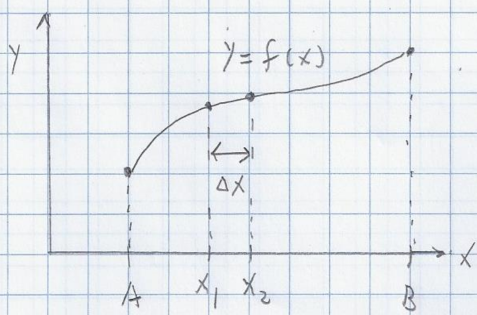
3.1. Numerical Differentiation



$A \equiv$ left endpoint
$B \equiv$ right endpoint
$N \equiv$ number of subdivisions

$\Delta X = \dfrac{B-A}{N} \equiv$ subdivision size

$X \in [A, B]$ is broken up into N subdivisions of length ΔX.

$$\boxed{f'\left(X_1 + \tfrac{1}{2}\Delta X\right) \approx \frac{f(X_2) - f(X_1)}{\Delta X}} \qquad (*)$$

Program ND (numerical differentiation)

We will type this program into the TI-84 as a class...

```
 PROGRAM:ND                             Comments
:ClrHome                                Erase the screen
:ClrList L1                             Erase list #1
:ClrList L2
:Disp "LEFT ENDPOINT:"                  Print LEFT ENDPOINT: to the screen
:Prompt A                               Ask for the value of A
:Disp " "                               Print a blank line on the screen
:Disp "RIGHT ENDPOINT:"
:Prompt B
:Disp " "
:Disp "NUMBER"
:Disp "SUBINTERVALS:"
:Prompt N
:(B-A)/N→G                              G ≡ ΔX
:For(I,1,N,1)                           I = 1, 2, ..., N
: A+(I-1)*G→D                           D ≡ X1
: D+G→E                                 E ≡ X2
: D+0.5*G→L1(I)                         D+0.5*G ≡ X1 + ½ΔX
: (Y1(E)-Y1(D))/G→L2(I)                 (Y1(X2)-Y1(X1))/ΔX   (Formula *)
:End
```

3.1. Numerical Differentiation

Example #1. Do as a class...

We want to differentiate numerically $f(x) = \sin x$.

(1) Put $\sin x$ into Y_1 of the "Y=" menu.

(2) Run program ND with

$A = 0$

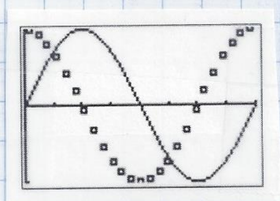
$B = 2*\pi$

$N = 25$

(3) Set your window to $x \in [0, 2\pi]$ and $y \in [-1, 1]$

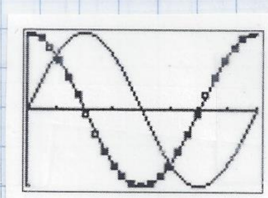
(4) GRAPH with STAT PLOTS 1: On

Your graph should look like...



The solid curve is $y = \sin x$. The plotted points are the numerical differentiation of $\sin x$, which you should notice looks like $y = \cos x$.

(5) Put $y = \cos x$ into Y_2 of the "Y=" menu and re-graph. Your graph should look like...

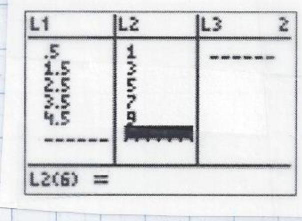


$y = \cos x$ runs through the plotted points $\Rightarrow$

$$\boxed{f(x) = \sin x \quad \Rightarrow \quad f'(x) = \cos x}$$

(6) Show me your graphs.

Example #2. If you run program ND with $Y_1 = x^2$, $A = 0$, $B = 5$ and $N = 5$, the lists L_1 and L_2 are ...



$(x, y) = (L_1, L_2)$ are points on the curve $y = f'(x)$ for $f(x) = x^2$, i.e., $f'(x) = 2x$. Note that the (x, y) values satisfy the derivative exactly $\Rightarrow$

The difference equation $(*)$ is exact for quadratic functions.

It is approximate otherwise.