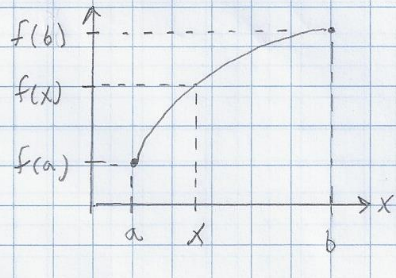
3.1. Endpoint Derivatives

Derivatives from the Left and Right. Note that the domain of $y = f(x)$ is $x \in [a, b]$.



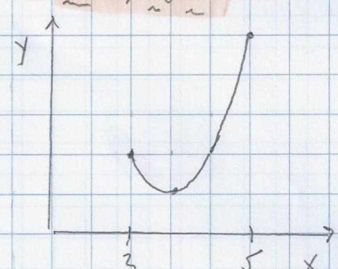
Definition of Derivative from the Right

$$\boxed{f'(a) = \lim_{x \to a^+} \frac{f(x) - f(a)}{x - a}}$$

Definition of Derivative from the Left

$$\boxed{f'(b) = \lim_{x \to b^-} \frac{f(b) - f(x)}{b - x}}$$

Example #1. For the function $f(x) = x^2 - 6x + 10$ defined on $x \in [2, 5]$ as graphed,



(a) calculate $f'(2)$ using the definition of derivative from the right

(b) Calculate $f'(5)$ using the definition of derivative from the left

(c) Calculate $f'(2)$ and $f'(5)$ by calculating $f'(x)$ by straightforward differentiation.

SOLUTION:

(a) $$f'(2) = \lim_{x \to 2^+} \frac{f(x) - f(2)}{x - 2} = \lim_{x \to 2^+} \frac{x^2 - 6x + 10 - 2}{x - 2} = \lim_{x \to 2^+} \frac{x^2 - 6x + 8}{x - 2} =$$

$$= \lim_{x \to 2^+} \frac{(\cancel{x-2})(x-4)}{\cancel{x-2}} = 2 - 4 = -2$$

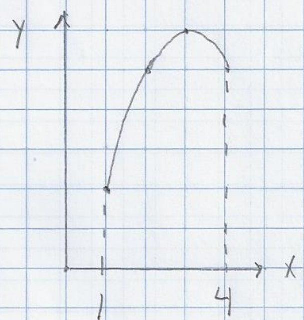
(b) $$f'(5) = \lim_{x \to 5^-} \frac{f(5) - f(x)}{5 - x} = \lim_{x \to 5^-} \frac{5 - (x^2 - 6x + 10)}{5 - x} = \lim_{x \to 5^-} \frac{-x^2 + 6x - 5}{5 - x} =$$

$$= \lim_{x \to 5^-} \frac{\cancel{(5 - x)}(x - 1)}{\cancel{5 - x}} = 5 - 1 = 4$$

(c) $f(x) = x^2 - 6x + 10 \Rightarrow f'(x) = 2x - 6$

$f'(2) = 2(2) - 6 = -2$ $\quad f'(5) = 2(5) - 6 = 4$

CLASS WORK. For $f(x) = -x^2 + 6x - 3$ defined on $x \in [1, 4]$ as graphed,



(a) calculate $f'(1)$ using the definition of derivative from the right

(b) calculate $f'(4)$ using the definition of derivative from the left

(c) calculate $f'(1)$ and $f'(4)$ by calculating $f'(x)$ by straightforward differentiation.

SOLUTION

(a) $$f'(1) = \lim_{x \to 1^+} \frac{f(x) - f(1)}{x - 1} = \lim_{x \to 1^+} \frac{-x^2 + 6x - 3 - 2}{x - 1} = \lim_{x \to 1^+} \frac{-x^2 + 6x - 5}{x - 1} =$$

$$= \lim_{x \to 1^+} \frac{\cancel{(x - 1)}(5 - x)}{\cancel{x - 1}} = 5 - 1 = 4$$

(b) $$f'(4) = \lim_{x \to 4^-} \frac{f(4) - f(x)}{4 - x} = \lim_{x \to 4^-} \frac{5 - (-x^2 + 6x - 3)}{4 - x} = \lim_{x \to 4^-} \frac{x^2 - 6x + 8}{4 - x} =$$

$$= \lim_{x \to 4^-} \frac{\cancel{(4 - x)}(2 - x)}{\cancel{4 - x}} = 2 - 4 = -2$$

(c) $f'(x) = -2x + 6$, $f'(1) = -2(1) + 6 = 4$

$f'(4) = -2(4) + 6 = -2$