AP CALCULUS AB

DIFFERENTIABILITY

For problems **1** through **3**:

a) Graph $y = f(x)$ on the grid provided.

b) State whether or not $y = f(x)$ is continuous at $x = 4$.

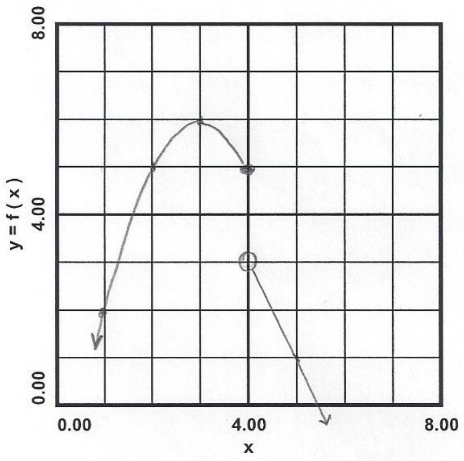
c) Use the (difference quotient) definitions of the derivatives from the left and the right to calculate $f'(4)$.

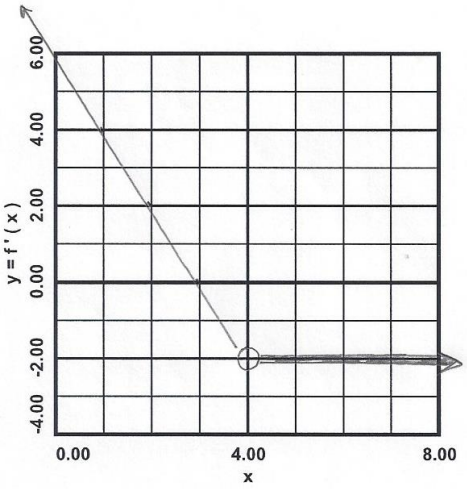
d) State whether or not $f'(4)$ exists and why.

e) Calculate $f'(x)$ by straightforward differentiation. Also, graph $y = f'(x)$ on the grid provided.

1)

$$f(x) = \begin{cases} -x^2 + 6x - 3 & , \quad -\infty < x \leq 4 \\ -2x + 11 & , \quad 4 < x < \infty \end{cases}$$





(b) $f(x)$ is not continuous at $x = 4$

(c) $\lim_{x \to 4^-} \frac{f(4) - f(x)}{4 - x} = \lim_{x \to 4^-} \frac{5 - (-x^2 + 6x - 3)}{4 - x} = \lim_{x \to 4^-} \frac{x^2 - 6x + 8}{4 - x} =$

$= \lim_{x \to 4^-} \frac{(4 - x)(2 - x)}{4 - x} = 2 - 4 = -2$ $\lim_{x \to 4^+} \frac{f(x) - f(4)}{x - 4} =$

$= \lim_{x \to 4^+} \frac{-2x + 11 - 5}{x - 4} = \lim_{x \to 4^+} \frac{-2x + 6}{x - 4} = \frac{-2}{0} = -\infty$

(d) $f'(4)$ does not exist because $f(x)$ is not continuous at $x = 4$.

(e) $f'(x) = \begin{cases} -2x + 6 & , \quad -\infty < x < 4 \\ -2 & , \quad 4 < x \leq \infty \end{cases}$

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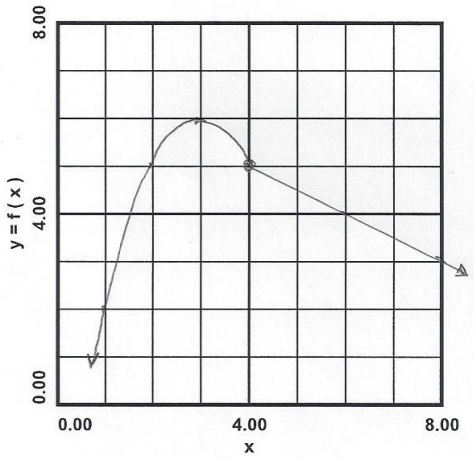
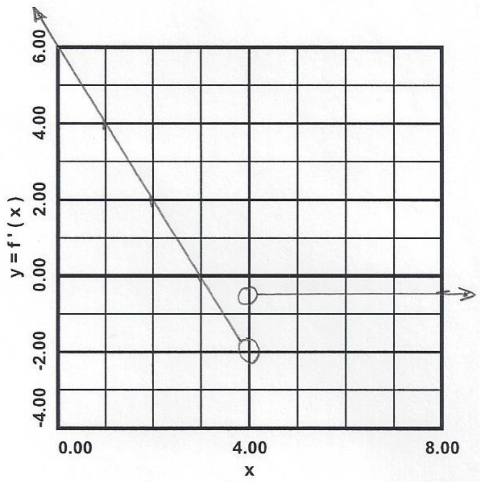
DIFFERENTIABILITY

2)

$$f(x) = \begin{cases} -x^2 + 6x - 3 & , \quad -\infty < x \leq 4 \\ -\frac{1}{2}x + 7 & , \quad 4 < x < \infty \end{cases}$$





(b) $f(x)$ is continuous at $x = 4$

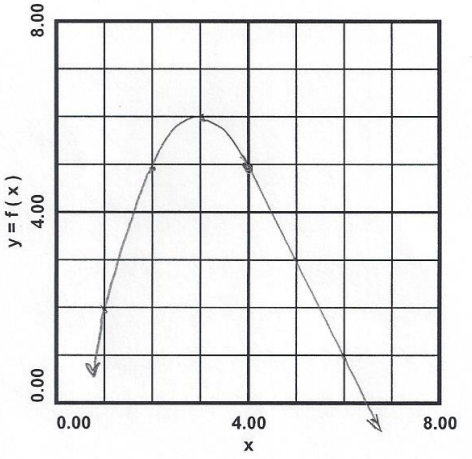
(c) See next page

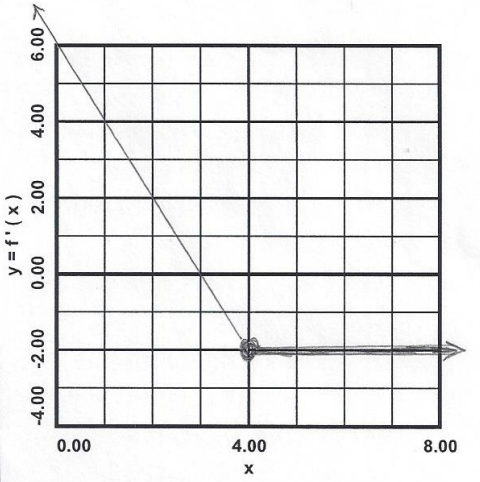
(d) $f'(4)$ does not exist because of the kink at $x = 4$

(e) $f'(x) = \begin{cases} -2x + 6, & -\infty < x < 4 \\ -\frac{1}{2}, & 4 < x < \infty \end{cases}$

3)

$$f(x) = \begin{cases} -x^2 + 6x - 3 & , \quad -\infty < x \leq 4 \\ -2x + 13 & , \quad 4 < x < \infty \end{cases}$$





(b) $f(x)$ is continuous at $x = 4$

(c) See next page

(d) $f'(4)$ does exist $\Leftarrow$ $f(x)$ is continuous and smooth at $x = 4$ (the left and right derivatives are both -2).

(e) $f'(x) = \begin{cases} -2x + 6, & -\infty < x \leq 4 \\ -2, & 4 < x < \infty \end{cases}$

Differentiability Worksheet

2c) $\lim_{x \to 4^-} \frac{f(4) - f(x)}{4 - x} = -2$ ✓

$\lim_{x \to 4^+} \frac{f(x) - f(4)}{x - 4} = \lim_{x \to 4^+} \frac{-\frac{1}{2}x + 7 - 5}{x - 4} = \lim_{x \to 4^+} \frac{-\frac{1}{2}x + 2}{x - 4} = \lim_{x \to 4^+} \frac{-\frac{1}{2}(x - 4)}{x - 4} = -\frac{1}{2}$ ✓

3c) $\lim_{x \to 4^-} \frac{f(4) - f(x)}{4 - x} = -2$ ✓

$\lim_{x \to 4^+} \frac{f(x) - f(4)}{x - 4} = \lim_{x \to 4^+} \frac{-2x + 13 - 5}{x - 4} = \lim_{x \to 4^+} \frac{-2x + 8}{x - 4} = \lim_{x \to 4^+} \frac{-2(x - 4)}{x - 4} = -2$ ✓