HW#3

3.1. Definition of Derivative

Supplimental:

(1)

```
          1
        1   1
      1   2   1
    1   3   3   1
  1   4   6   4   1
1   5  10  10   5   1
```

$f(x+h) = (x+h)^5 =$

$= \underbrace{x^5}_{f(x)} + 5x^4h + 10x^3h^2 + 10x^2h^3 + 5xh^4 + h^5$

$f(x+h) - f(x) = 5x^4h + 10x^3h^2 + 10x^2h^3 + 5xh^4 + h^5$

$\frac{f(x+h) - f(x)}{h} = 5x^4 + 10x^3h + 10x^2h^2 + 5xh^3 + h^4$

$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = 5x^4$ ◀

(2) $f(x+h) - f(x) = \frac{1}{(x+h)^3} - \frac{1}{x^3} = \frac{1}{(x+h)^3} \cdot \frac{x^3}{x^3} - \frac{1}{x^3} \cdot \frac{(x+h)^3}{(x+h)^3} =$

$= \frac{x^3 - (x+h)^3}{(x+h)^3x^3} = \frac{x^3 - (x^3 + 3x^2h + 3xh^2 + h^3)}{(x+h)^3x^3} = \frac{-3x^2h - 3xh^2 - h^3}{(x+h)^3x^3}$

$\frac{f(x+h) - f(x)}{h} = \frac{-3x^2 - 3xh - h^2}{(x+h)^3x^3}, \quad \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \frac{-3x^2}{x^3 \cdot x^3} = -\frac{3}{x^4}$ ◀

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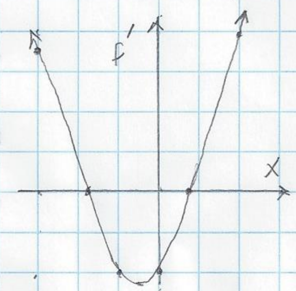
(13) b ◀ (14) a ◀ (15) d ◀ (16) c ◀

(22) Estimated slopes from the graph

x	f′
−3	3.5
−1.8	0
−1	−2
0	−2
0.8	0
2	4



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3.3. Power Rule

(1) $f(x) = -x^2 + 3, \quad f'(x) = -2x$

(3) $f(x) = 2x + 1, \quad f'(x) = 2$

(5) $f(x) = \frac{x^3}{3} + \frac{x^2}{2} + x, \quad f'(x) = \frac{1}{3}\cdot 3x^2 + \frac{1}{2}\cdot 2x + 1 = x^2 + x + 1$

(7) $f(x) = x^3 - 2x^2 + x + 1, \quad f'(x) = 3x^2 - 4x + 1 \quad ac = 3 = tu \quad b = -4 = t + u$

$t = -3 \quad u = -1 \quad 3x^2 - 3x - x + 1 = 3x(x-1) - 1\cdot(x-1) = (3x-1)(x-1)$

$f'(x) = (3x-1)(x-1) = 0 \Rightarrow x = \frac{1}{3}$ and $x = 1$

(9) $f(x) = x^4 - 4x^2 + 1, \quad f'(x) = 4x^3 - 8x = 4x(x^2 - 2) = 0 \Rightarrow$

$x = 0$ and $x = \pm\sqrt{2}$

(11) $f(x) = 5x^3 - 3x^5, \quad f'(x) = 15x^2 - 15x^4 = 15x^2(1 - x^2) = 0 \Rightarrow$

$x = 0$ and $x = \pm 1$

(29) $f(x) = 4x^{-2} - 8x + 1, \quad f'(x) = -8x^{-3} - 8 = -\frac{8}{x^3} - 8$

(30) $f(x) = \frac{1}{4}x^{-4} - \frac{1}{3}x^{-3} + \frac{1}{2}x^{-2} - x^{-1} + 3$

$f'(x) = \frac{1}{4}\cdot -4x^{-5} - \frac{1}{3}\cdot -3x^{-4} + \frac{1}{2}\cdot -2x^{-3} - -1x^{-2} = -\frac{1}{x^5} + \frac{1}{x^4} - \frac{1}{x^3} + \frac{1}{x^2}$

Supplemental: 3.3. Irrational and Rational Powers

(3) $f(x) = 7x^{3/4} - 3x^{-4/5}, \quad f'(x) = 7\cdot\frac{3}{4}x^{3/4-1} - 3\cdot -\frac{4}{5}x^{-4/5-1} =$

$= \frac{21}{4}x^{-1/4} + \frac{12}{5}x^{-9/5} = \frac{21}{4\cdot\sqrt[4]{x}} + \frac{12}{5\cdot\sqrt[5]{x^9}}$

(4) $f(x) = 11x^{7/3} - 2x^{-9/5}, \quad f'(x) = 11\cdot\frac{7}{3}x^{7/3-1} - 2\cdot -\frac{9}{5}x^{-9/5-1} =$

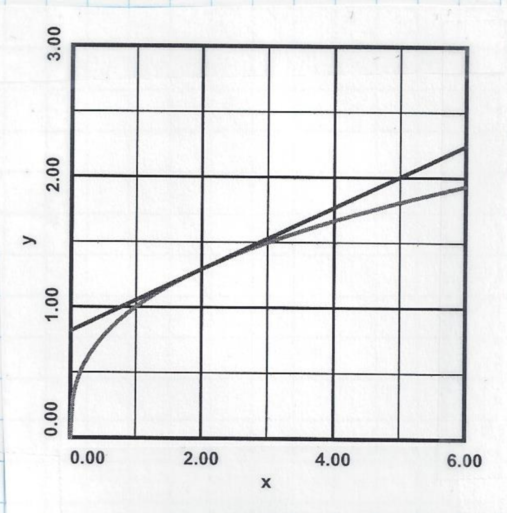
$= \frac{77}{3}x^{4/3} + \frac{18}{5}x^{-14/5} = \frac{77\cdot\sqrt[3]{x^4}}{3} + \frac{18}{5\cdot\sqrt[5]{x^{14}}}$

(5) $f(x) = x^{\frac{1}{e}}$, $f'(x) = \frac{1}{e} x^{\frac{1}{e}-1}$, $f(2) = 2^{1/e}$, $f'(2) = \frac{1}{e} \cdot 2^{1/e - 1} = \frac{2^{1/e}}{2e}$

(a) $y = \frac{2^{1/e}}{2e} x + b$, $2^{1/e} = \frac{2^{1/e}}{2e} \cdot 2 + b$, $b = \frac{2^{1/e}}{e}(e-1)$

$y = \frac{2^{1/e}}{2e} x + \frac{2^{1/e}}{e}(e-1) = \frac{2^{1/e}}{2e}(x + 2e - 2)$

(b)

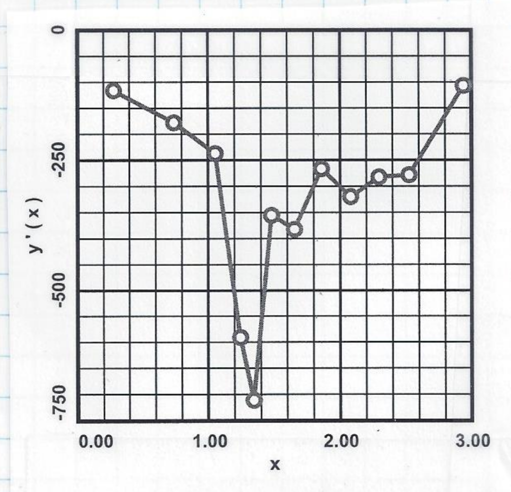


pg 107 3.1. Numerical Differentiation

(30)

(b)

x	$f'(x)$
0.28	−116.1
0.74	−177.8
1.055	−237.0
1.245	−590.9
1.345	−711.1
1.48	−355.6
1.655	−382.4
1.86	−266.7
2.08	−320.0
2.295	−282.6
2.525	−278.3
2.94	−106.7



(c) $\frac{ft}{mi}$ (d) $\frac{ft}{mi}$

(f) The most dangerous section is at $x = 1.345$ mi, which is where the magnitude of $y'(x)$ is greatest, i.e., where the river is dropping the fastest

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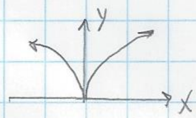
(17) $f'(0) \approx \dfrac{f(0.0005) - f(-0.0005)}{0.001} = 4$ which is the exact result.

Is differentiable at $x = 0$

(21) $f'(-2) \approx \dfrac{f(-1.9995) - f(-2.0005)}{0.001} = 8.000\,000\,25$ The exact result is 8. Is differentiable at $x = -2$

(25) $f'(0) \approx \dfrac{f(0.0005) - f(-0.0005)}{0.001} = 0$ which is not correct.



Not differentiable at $x=0$ due to the cusp

Pg. 107 3.1. Endpoint Derivatives

DL ≡ Derivative from the left, DR ≡ Derivative from the right

(31) $f(x) = \begin{bmatrix} x^2 + x, & x \le 1 \\ 3x - 2, & x > 1 \end{bmatrix}$ $LD(1) = \lim\limits_{x \to 1^-} \dfrac{f(1) - f(x)}{1 - x} = \lim\limits_{x \to 1^-} \dfrac{2 - (x^2 + x)}{1 - x} =$

$= \lim\limits_{x \to 1^-} \dfrac{-x^2 - x + 2}{1 - x} = \lim\limits_{x \to 1^-} \dfrac{(x+2)\cancel{(1-x)}}{\cancel{1-x}} = 3$ $RD(1) = \lim\limits_{x \to 1^+} \dfrac{f(x) - f(1)}{x - 1} =$

$= \lim\limits_{x \to 1^+} \dfrac{(3x-2) - 2}{x - 1} = \lim\limits_{x \to 1^+} \dfrac{3x - 4}{x - 1} = \dfrac{-1}{0^+} = -\infty$ $LD(1) \ne RD(1) \Rightarrow f'(1)$ D.N.E.

(32) $f(x) = \begin{bmatrix} x^3, & x \le 1 \\ 3x, & x > 1 \end{bmatrix}$ $LD(1) = \lim\limits_{x \to 1^-} \dfrac{f(1) - f(x)}{1 - x} = \lim\limits_{x \to 1^-} \dfrac{1 - x^3}{1 - x} =$

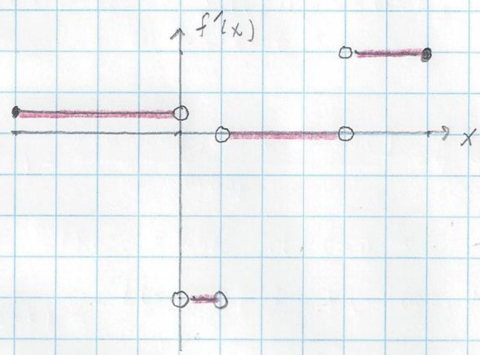
$= \lim\limits_{x \to 1^-} \dfrac{\cancel{(1-x)}(x^2 + x + 1)}{\cancel{1-x}} = 3$ $RD(1) = \lim\limits_{x \to 1^+} \dfrac{f(x) - f(1)}{x - 1} =$

$$= \lim_{x \to 1^+} \frac{3x-1}{x-1} = \frac{2}{0^+} = \infty \quad LD(1) \neq RD(1) \Rightarrow f'(1) \text{ D.N.E.}$$

pg. 107 3.2. Differentiability

(26)

(a)



(b) not differentiable at $x = 0$, 1 and 4

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(5) (a) $x \in [-3, 2]$ (b) $x \in \phi$ (c) $x \in \phi$

(7) (a) $x \in [-3, 0) \cup (0, 3]$ (b) $x \in \phi$ (c) $x = 0$

(9) (a) $x \in [-1, 0) \cup (0, 2]$ (b) $x = 0$ (c) $x \in \phi$

(31) $f(x) = \dfrac{x^3 - 8}{x^2 - 4x - 5} = \dfrac{x^3 - 8}{(x-5)(x+1)} \Rightarrow$ differentiable for $x \in (-\infty, -1) \cup (-1, 5) \cup (5, \infty)$

(33) $f(x) = \sin|x| - 1$, differentiable for $x \in (-\infty, 0) \cup (0, \infty)$

There is a kink at $x = 0$.

(35)
$$g(x) = \begin{bmatrix} (x+1)^2, & x \leq 0 \\ 2x+1, & 0 < x < 3 \\ (4-x)^2, & x \geq 3 \end{bmatrix}$$

continuous at $x = 0$
not continuous at $x = 3$

$$g'(x) = \begin{bmatrix} 2x+2, & x \leq 0 \\ 2, & 0 < x < 3 \\ -8+2x, & x > 3 \end{bmatrix}$$

g' is continuous at $x = 0$
g' not defined at $x = 3$

so, differentiable for $x \in (-\infty, 3) \cup (3, \infty)$