Quiz #3 Study Guide

(1) $f(x+h)-f(x) = \frac{1}{(x+h)^3} - \frac{1}{x^3} = \frac{1}{(x+h)^3}\cdot\frac{x^3}{x^3} - \frac{1}{x^3}\cdot\frac{(x+h)^3}{(x+h)^3} =$

$= \frac{x^3-(x+h)^3}{(x+h)^3x^3} = \frac{x^3-(x^3+3x^2h+3xh^2+h^3)}{(x+h)^3x^3} = \frac{-3x^2h-3xh^2-h^3}{(x+h)^3x^3}$

$m_s(x,h) = \frac{f(x+h)-f(x)}{h} = \frac{-3x^2-3xh-h^2}{(x+h)^3x^3}$

$m_T(x) = \lim_{h\to 0} m_s(x,h) = \frac{-3x^2}{x^3\cdot x^3} = -\frac{3}{x^4}$

(2) $f'(x) = 5\cdot 3x^2+13\cdot 2x-11 = 15x^2+26x-11$

(3) $f(x)=4x^{-5}-6x^{-7}$, $f'(x) = 4\cdot -5x^{-6}-6\cdot -7x^{-8} = -\frac{20}{x^6}+\frac{42}{x^8}$

(4) $f(x)=13x^{11/9}-14x^{-4/7}$, $f'(x)=13\cdot\frac{11}{9}x^{11/9-1}-14\cdot -\frac{4}{7}x^{-4/7-1}=$

$=\frac{143}{9}x^{2/9}+8x^{-11/7} = \frac{143\cdot\sqrt[9]{x^2}}{9}+\frac{8}{\sqrt[7]{x^{11}}}$

(5) $f'(x)=6x^2-2x-48=0$, $3x^2-x-24=0$ $\quad ac=3(-24)=-72=-2^3\cdot 3^2=tu$

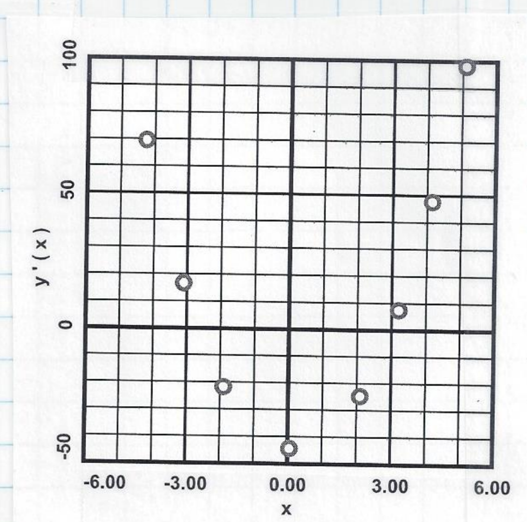
$b=-1=t+u$ $\quad t=-9$ $\quad u=8$ $\quad 3x^2-9x+8x-24 = 3x(x-3)+8(x-3)=$

$=(3x+8)(x-3) \Rightarrow x=-\frac{8}{3}$ and $x=3$

(6)
(a) $\Delta x = x_2-x_1$ $\quad y'(x_1+\frac{\Delta x}{2}) \approx \frac{y(x_2)-y(x_1)}{\Delta x} \Rightarrow$

x	y'(x)
-4.25	69.5
-3.10	16.6
-1.90	-21.8
0.05	-44.4
2.10	-24.8
3.20	7.4
4.15	47.6
5.10	98.1

(b)



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(7) $LD(3) = \lim_{x \to 3^-} \frac{f(3) - f(x)}{3 - x} = \lim_{x \to 3^-} \frac{4 - (x^2 - 4x + 6)}{3 - x} = \lim_{x \to 3^-} \frac{-x^2 + 4x - 2}{3 - x} =$

$= \frac{1}{0^+} = \infty$ ◄

$RD(3) = \lim_{x \to 3^+} \frac{f(x) - f(3)}{x - 3} = \lim_{x \to 3^+} \frac{(-x^2 + 8x - 11) - 4}{x - 3} = \lim_{x \to 3^+} \frac{-x^2 + 8x - 15}{x - 3} =$

$= \lim_{x \to 3^+} \frac{(5 - x)\cancel{(x - 3)}}{\cancel{x - 3}} = 5 - 3 = 2$ ◄

(8) This is the same function as in #7. $LD(3) \neq RD(3) \Rightarrow$
not differentiable at $x = 3$.
So, is differentiable for $x \in (-\infty, 3) \cup (3, \infty)$ ◄

(9) $f(x)$ is always positive & smooth $\Rightarrow x \in (-\infty, \infty)$ ◄

(10) $4x^2 + 7x - 15 = 4x^2 + 12x - 5x - 15 = 4x(x + 3) - 5(x + 3) = (4x - 5)(x + 3)$

$ac = 4(-15) = -60 = -2^2 \cdot 3 \cdot 5 = tu \quad b = 7 = t + u \quad t = 12 \quad u = -5$

$f(x) = \frac{x + 8}{(4x - 5)(x + 3)} \Rightarrow$ V.A.'s at $x = \frac{5}{4}$ and $x = -3 \Rightarrow$

$x \in (-\infty, -3) \cup (-3, \frac{5}{4}) \cup (\frac{5}{4}, \infty)$ ◄