

3.3. Higher-order Derivatives

10F 2

Notation

Newton's Notation

$f'(x) \equiv$ the derivative of $f(x)$

$f''(x) \equiv$ the second derivative of $f(x)$
 \equiv the derivative of $f'(x)$

$f'''(x) \equiv$ the third derivative of $f(x)$
 \equiv the derivative of $f''(x)$

Leibnitz's Notation

$$\frac{df}{dx}$$

$$\frac{d^2f}{dx^2}$$

$$\frac{d^3f}{dx^3}$$

Example #1. Calculate the 3rd derivative of

$$f(x) = \frac{1}{72}x^9 + \frac{1}{42}x^7 + \frac{1}{20}x^5.$$

SOLUTION:

$$f'(x) = \frac{9}{72}x^8 + \frac{7}{42}x^6 + \frac{5}{20}x^4 = \frac{1}{8}x^8 + \frac{1}{6}x^6 + \frac{1}{4}x^4$$

$$f''(x) = \frac{8}{8}x^7 + \frac{6}{6}x^5 + \frac{4}{4}x^3 = x^7 + x^5 + x^3$$

$$f'''(x) = 7x^6 + 5x^4 + 3x^2 \quad \blacktriangleleft$$

Example #2. Calculate the 2nd derivative of

$$f(x) = \sqrt[4]{x^{13}} + \frac{1}{7\sqrt{x^8}}.$$

SOLUTION:

$$f(x) = x^{13/4} + x^{-8/7} \quad f'(x) = \frac{13}{4}x^{13/4-1} - \frac{8}{7}x^{-8/7-1} = \frac{13}{4}x^{9/4} - \frac{8}{7}x^{-15/7}$$

$$f''(x) = \frac{13}{4} \cdot \frac{9}{4}x^{9/4-1} - \frac{8}{7} \cdot \frac{-15}{7}x^{-15/7-1} = \frac{117}{16}x^{5/4} + \frac{120}{49}x^{-22/7}$$

$$f''(x) = \frac{117 \cdot \sqrt[4]{x^5}}{16} + \frac{120}{49 \cdot 7\sqrt{x^{22}}} \quad \blacktriangleleft$$

3.3. Higher - order derivatives

20F 2

CLASS WORK

- 1) Calculate the 3rd derivative of $f(x) = \frac{1}{504}x^9 + \frac{1}{210}x^7 + \frac{1}{60}x^5$.
- 2) Calculate the 2nd derivative of $f(x) = \sqrt[5]{x^{18}} + \frac{1}{\sqrt[6]{x^{11}}}$.

SOLUTIONS

1) $f'(x) = \frac{9}{504}x^8 + \frac{7}{210}x^6 + \frac{5}{60}x^4 = \frac{1}{56}x^8 + \frac{1}{30}x^6 + \frac{1}{12}x^4$

$$f''(x) = \frac{8}{56}x^7 + \frac{6}{30}x^5 + \frac{4}{12}x^3 = \frac{1}{7}x^7 + \frac{1}{5}x^5 + \frac{1}{3}x^3$$

$$f'''(x) = x^6 + x^4 + x^2 \quad \blacktriangleleft$$

2) $f(x) = x^{\frac{18}{5}} + x^{-\frac{11}{6}}$, $f'(x) = \frac{18}{5}x^{\frac{18}{5}-1} - \frac{11}{6}x^{-\frac{11}{6}-1} = \frac{18}{5}x^{\frac{13}{5}} - \frac{11}{6}x^{-\frac{17}{6}}$

$$f''(x) = \frac{18}{5} \cdot \frac{13}{5}x^{\frac{13}{5}-1} - \frac{11}{6} \cdot -\frac{17}{6}x^{-\frac{17}{6}-1} = \frac{234}{25}x^{\frac{8}{5}} + \frac{187}{36}x^{-\frac{23}{6}}$$

$$f''(x) = \frac{234 \cdot \sqrt[5]{x^8}}{25} + \frac{187}{36 \cdot \sqrt[6]{x^{23}}} \quad \blacktriangleleft$$