3.3. Higher-Order Derivatives

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(33) $y = x^4 + x^3 - 2x^2 + x - 5$, $\frac{dy}{dx} = 4x^3 + 3x^2 - 4x + 1$

$\frac{d^2y}{dx^2} = 12x^2 + 6x - 4$ $\frac{d^3y}{dx^3} = 24x + 6$ $\frac{d^4y}{dx^4} = 24$

(36) $y = \frac{x+1}{x} = 1 + \frac{1}{x} = 1 + x^{-1}$ $\frac{dy}{dx} = -1x^{-2} = -\frac{1}{x^2}$

$\frac{d^2y}{dx^2} = -1 \cdot -2x^{-3} = 2x^{-3} = \frac{2}{x^3}$ $\frac{d^3y}{dx^3} = 2 \cdot -3x^{-4} = -6x^{-4} = -\frac{6}{x^4}$

$\frac{d^4y}{dx^4} = -6 \cdot -4x^{-5} = 24x^{-5} = \frac{24}{x^5}$

3.3. Product and Quotient Rules

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(13) $y = (x+1)(x^2+1) = x^3 + x + x^2 + 1 = x^3 + x^2 + x + 1$

(a) $\frac{dy}{dx} = 1 \cdot (x^2+1) + (x+1)(2x) = x^2 + 1 + 2x^2 + 2x = 3x^2 + 2x + 1$

(b) $\frac{dy}{dx} = 3x^2 + 2x + 1$

(14) $y = \frac{x^2+3}{x} = \frac{u}{v} = x + \frac{3}{x} = x + 3x^{-1}$

(a) $\frac{dy}{dx} = \frac{u'v - uv'}{v^2} = \frac{2x \cdot x - (x^2+3) \cdot 1}{x^2} = \frac{2x^2 - x^2 - 3}{x^2} = \frac{x^2-3}{x^2} = 1 - \frac{3}{x^2}$

(b) $\frac{dy}{dx} = 1 + 3(-1)x^{-2} = 1 - \frac{3}{x^2}$

(15) $y = (x^3+x+1)(x^4+x^2+1)$, $\frac{dy}{dx} = (3x^2+1)(x^4+x^2+1) + (x^3+x+1)(4x^3+2x)$

(17) $y = \frac{2x+5}{3x-2} = \frac{u}{v}$ $\frac{dy}{dx} = \frac{u'v - uv'}{v^2} = \frac{2(3x-2) - (2x+5)(3)}{(3x-2)^2}$

(19) $y = \frac{(x-1)(x^2+x+1)}{x^3} = \frac{u}{v}$ $u' = 1(x^2+x+1) + (x-1)(2x+1)$

$\frac{dy}{dx} = \frac{u'v - uv'}{v^2} = \frac{[x^2+x+1+(x-1)(2x+1)]x^3 - (x-1)(x^2+x+1) \cdot 3x^2}{x^6}$

(22) $y = \dfrac{(x+1)(x+2)}{(x-1)(x-2)} = \dfrac{u}{v}$ $u' = 1(x+2) + (x+1)1 = x+2+x+1 = 2x+3$

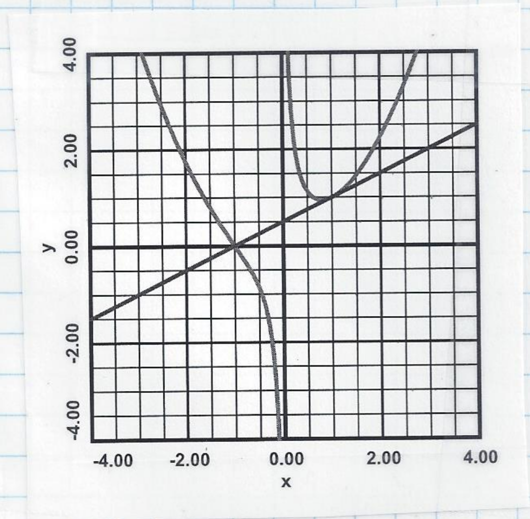
$v' = 1(x-2) + (x-1)1 = x-2+x-1 = 2x-3$

$$\frac{dy}{dx} = \frac{u'v - uv'}{v^2} = \frac{(2x+3)(x-1)(x-2) - (x+1)(x+2)(2x-3)}{(x-1)^2(x-2)^2}$$

(27) $y = \dfrac{x^3+1}{2x} = \dfrac{u}{v}$ $\dfrac{dy}{dx} = \dfrac{u'v - uv'}{v^2} = \dfrac{(3x^2)(2x) - (x^3+1)(2)}{4x^2} = \dfrac{6x^3 - 2x^3 - 2}{4x^2} =$

$= \dfrac{4x^3-2}{4x^2} = \dfrac{2x^3-1}{2x^2}$ $y(1) = 1$ $y'(1) = \frac{1}{2}$

$y = \frac{1}{2}x + b$ $1 = \frac{1}{2}(1) + b$ $b = \frac{1}{2}$ $y = \frac{1}{2}x + \frac{1}{2}$



(45) $\dfrac{d}{dx}\left[\dfrac{1}{f(x)}\right] = \dfrac{d}{dx}\left[\dfrac{u}{v}\right] = \dfrac{u'v - uv'}{v^2} =$

$= \dfrac{0 \cdot f - 1 \cdot f'}{f^2} = -\dfrac{f'(x)}{f^2(x)}$

(46) $P = \dfrac{nRT}{V-nb} - \dfrac{an^2}{V^2}$

$\dfrac{d}{dV}\left[\dfrac{nRT}{V-nb}\right] = \dfrac{d}{dV}\left[\dfrac{u}{v}\right] = \dfrac{u'v - uv'}{v^2} =$

$= \dfrac{0(V-nb) - nRT(1)}{(V-nb)^2} = -\dfrac{nRT}{(V-nb)^2}$

$\dfrac{d}{dV}\left[\dfrac{an^2}{V^2}\right] = \dfrac{d}{dV}\left[an^2V^{-2}\right] = an^2 \cdot -2V^{-3} = -\dfrac{2an^2}{V^3} \Rightarrow$

$\dfrac{dP}{dV} = -\dfrac{nRT}{(V-nb)^2} - \dfrac{2an^2}{V^3}$

Pg. 137 3.4. Equations of Motion

(8) $Q(t) = 200(30-t)^2 = 200(900 - 60t + t^2)$, $Q'(t) = 200(-60 + 2t) =$

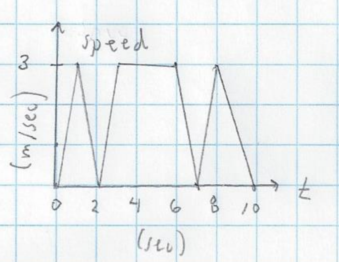
$= -12{,}000 + 400t$. $Q'(10) = -8000 \frac{gal}{min}$

$\dfrac{Q(10) - Q(0)}{10-0} = \dfrac{80{,}000 - 180{,}000}{10-0} = -10{,}000 \frac{gal}{min}$

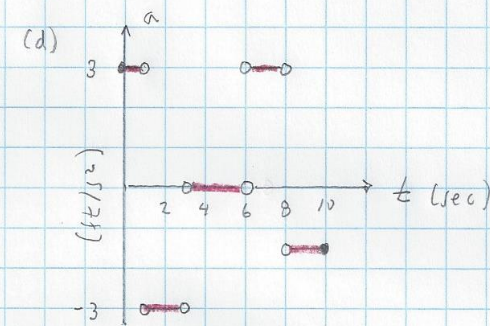
(11) (a) $V=0 \Rightarrow$ reverses direction at $t=2$ and 7 sec ◄

(b) $t \in (3,6)$ sec ◄

(c)



(d)



(13) (a) $s = 24t - 0.8t^2 \quad V = \frac{ds}{dt} = 24 - 1.6t$ ◄ $\quad a = \frac{dv}{dt} = -1.6$ ◄

(b) $V=0 \Rightarrow 0 = 24 - 1.6t, \; t = 15$ sec ◄

(c) $s(15) = 180$ m ◄

(d) $90 = 24t - 0.8t^2, \quad 0.8t^2 - 24t + 90 = 0 \quad t = \frac{24 \pm \sqrt{24^2 - 4(0.8)(90)}}{2(0.8)} = \frac{24 \pm \sqrt{288}}{1.6}$

$t = 4.393$ sec ◄ on way up

$t = 25.607$ sec ◄ on way down

(e) $s = 0 = 24t - 0.8t^2 = t(24 - 0.8t) \Rightarrow t = 30$ sec ◄

(18) (a) $190 \frac{ft}{sec}$ ◄ (b) 2 sec ◄ (c) $t = 8$ sec $(V=0)$ ◄

(d) $t = 10.8$ sec $(90 \frac{ft}{sec})$ ◄ (e) $10.8 - 8 = 2.8$ sec ◄

(f) $t = 2$ sec is greatest acceleration ◄

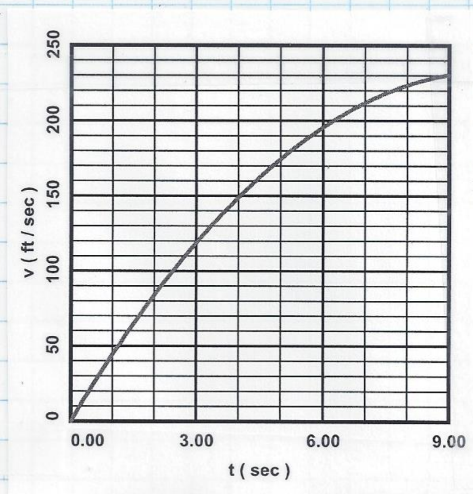
acceleration is constant during $t \in (2, 10.8)$ sec ◄

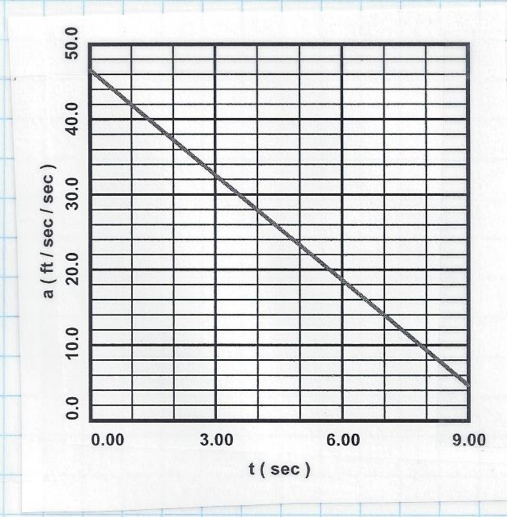
Supplimental

(1) (a) $x = \frac{440}{567}(30t^2 - t^3) \quad v = \frac{dx}{dt} = \frac{440}{567}(60t - 3t^2) = \frac{440}{189}(20t - t^2)$ ◄

$a = \frac{dv}{dt} = \frac{440}{189}(20 - 2t) = \frac{880}{189}(10 - t)$ ◄

(b)





(c) $V_{max} = V(9) = 230.48\ \frac{ft}{sec}$ ◄ $a_{max} = a(0) = 46.56\ \frac{ft}{sec^2}$ ◄

Supplimental 3.5. Two Important Limits

(2) $\lim_{\theta \to 0} \frac{\sin 2\theta - 2\sin\theta}{\theta} = \lim_{\theta \to 0} \frac{2\sin\theta\cos\theta - 2\sin\theta}{\theta} = 2 \lim_{\theta \to 0} \left(\frac{\sin\theta}{\theta}\right) \cdot \left(\cos\theta - 1\right) =$

$= 2 \cdot 1 \cdot (1-1) = 0$ ◄

(3) $\lim_{\theta \to 0} \frac{\sin 2\theta}{\theta} = \lim_{\theta \to 0} \frac{2\sin\theta\cos\theta}{\theta} = 2 \lim_{\theta \to 0} \left(\frac{\sin\theta}{\theta}\right)\left(\cos\theta\right) = 2 \cdot 1 \cdot 1 = 2$ ◄

pg. 147 3.5. Derivatives of the Trigonometric Functions

(1) $y = 1 + x - \cos x \quad \frac{dy}{dx} = 1 - -\sin x = 1 + \sin x$ ◄

(3) $y = \frac{1}{x} + 5\sin x = x^{-1} + 5\sin x \quad \frac{dy}{dx} = -1x^{-2} + 5 \cdot \cos x = -\frac{1}{x^2} + 5\cos x$ ◄

(5) $y = 4 - x^2 \sin x \quad \frac{dy}{dx} = -\left[2x\sin x + x^2\cos x\right] = -2x\sin x - x^2\cos x$ ◄

(7) $y = \frac{4}{\cos x} = 4\sec x \quad \frac{dy}{dx} = 4\sec x\tan x$ ◄

HW#4

(9) $y = \dfrac{\cot x}{1+\cot x} = \dfrac{u}{v}$ $\quad \dfrac{dy}{dx} = \dfrac{u'v - uv'}{v^2} = \dfrac{-\csc^2 x\,(1+\cot x) - \cot x\,(-\csc^2 x)}{(1+\cot x)^2} =$

$= -\dfrac{\csc^2 x}{(1+\cot x)^2}$ ◄

(27) $y = \sec x \quad \dfrac{dy}{dx} = \sec x \tan x \quad \left.\dfrac{dy}{dx}\right|_{x=0} = 1 \cdot 0 = 0$ ◄

$y = \cos x \quad \dfrac{dy}{dx} = -\sin x \quad \left.\dfrac{dy}{dx}\right|_{x=0} = -0 = 0$ ◄

(28) $y = \tan x \quad \dfrac{dy}{dx} = \sec^2 x \neq 0$ because $\sec^2 x \geq 1$ ◄

$y = \cot x \quad \dfrac{dy}{dx} = -\csc^2 x \neq 0$ because $\csc^2 x \geq 1$ ◄

(42) $y = \sin 2x = 2\sin x \cos x$, $\quad y' = 2\left[\cos x \cdot \cos x + \sin x \cdot -\sin x\right] =$

$= 2\left[\cos^2 x - \sin^2 x\right] = 2\cos 2x$ ◄

(43) $y = \cos 2x = \cos x \cdot \cos x - \sin x \cdot \sin x$

$y' = \left[-\sin x \cdot \cos x + \cos x \cdot -\sin x\right] - \left[\cos x \cdot \sin x + \sin x \cdot \cos x\right] =$

$= -\sin x \cos x - \sin x \cos x - \sin x \cos x - \sin x \cos x = -4\sin x \cos x =$

$= -2(2\sin x \cos x) = -2\sin 2x$ ◄