CHAPTER 4 Review Exercises

Exercise numbers with a gray background indicate problems that the authors have designed to be solved *without a calculator.*

The collection of exercises marked in red could be used as a chapter test.

In Exercises 1–30, find the derivative of the function.

1. $y = e^{3x-7}$

2. $y = \tan(e^x)$

3. $y = \sin^3 x$

4. $y = \ln(\csc x)$

5. $s = \cos(1 - 2t)$

6. $s = \cot \dfrac{2}{t}$

7. $y = \sqrt{1 + \cos x}$

8. $y = x\sqrt{2x + 1}$

9. $r = \sec(1 + 3\theta)$

10. $r = \tan^2(3 - \theta^2)$

11. $y = x^2 \csc 5x$

12. $y = \ln \sqrt{x}$

13. $y = \ln(1 + e^x)$

14. $y = xe^{-x}$

15. $y = e^{(1+\ln x)}$

16. $y = \ln(\sin x)$

17. $r = \ln(\cos^{-1} x)$

18. $r = \log_2(\theta^2)$

19. $s = \log_5(t - 7)$

20. $s = 8^{-t}$

21. $y = x^{\ln x}$

22. $y = \dfrac{(2x)2^x}{\sqrt{x^2 + 1}}$

23. $y = e^{\tan^{-1} x}$

24. $y = \sin^{-1}\sqrt{1 - u^2}$

25. $y = t \sec^{-1} t - \dfrac{1}{2}\ln t$

26. $y = (1 + t^2)\cot^{-1} 2t$

27. $y = z\cos^{-1} z - \sqrt{1 - z^2}$

28. $y = 2\sqrt{x - 1}\csc^{-1}\sqrt{x}$

29. $y = \csc^{-1}(\sec x),\ 0 \le x < \dfrac{\pi}{2}$

30. $r = \left(\dfrac{1 + \sin\theta}{1 - \cos\theta}\right)^2$

In Exercises 31–34, find all values of x for which the function is differentiable.

31. $y = \ln x^2$

32. $y = \sin(e^{2x})$

33. $y = \sqrt{\dfrac{1 - x}{1 + x^2}}$

34. $y = \dfrac{1}{1 - e^x}$

In Exercises 35–38, find dy/dx.

35. $xy + 2x + 3y = 1$

36. $5x^{4/5} + 10y^{6/5} = 15$

37. $\sqrt{xy} = 1$

38. $y^2 = \dfrac{x}{x + 1}$

In Exercises 39–42, find d^2y/dx^2 by implicit differentiation.

39. $x^3 + y^3 = 1$

40. $y^2 = 1 - \dfrac{2}{x}$

41. $y^3 + y = 2\cos x$

42. $x^{1/3} + y^{1/3} = 4$

In Exercises 43 and 44, find $\dfrac{d^{40}y}{dx^{40}}$.

43. $y = e^{x\sqrt[4]{2}}$

44. $y = \sin(x\sqrt[8]{2})$

In Exercises 45–48, find an equation for the **(a)** tangent and **(b)** normal to the curve at the indicated point.

45. $y = \sqrt{x^2 - 2x},\quad x = 3$

46. $y = \tan 2x,\quad x = \pi/3$

47. $x^2 + 2y^2 = 9,\quad (1, 2)$

48. $x + \sqrt{xy} = 6,\quad (4, 1)$

In Exercises 49–52, find an equation for the line tangent to the curve at the point defined by the given value of t.

49. $x = 2\sin t,\quad y = 2\cos t,\quad t = 3\pi/4$

50. $x = 3\cos t,\quad y = 4\sin t,\quad t = 3\pi/4$

51. $x = 3\sec t,\quad y = 5\tan t,\quad t = \pi/6$

52. $x = \cos t,\quad y = t + \sin t,\quad t = -\pi/4$

53. Writing to Learn

Let $f(x) = \begin{cases} \sin ax + b\cos x, & x < 0 \\ 5x + 3, & x \ge 0 \end{cases}$.

(a) If f is continuous at $x = 0$, find the value of b. Justify your answer.

(b) If f is differentiable at $x = 0$, find the value of a. Justify your answer.

(c) Is f differentiable at $x = 0$ if $a = 5$ and $b = 4$? Justify your answer.

54. Writing to Learn For what values of the constant m is

$$f(x) = \begin{cases} \sin 2x, & x \le 0 \\ mx, & x > 0 \end{cases}$$

(a) continuous at $x = 0$? Explain.

(b) differentiable at $x = 0$? Explain.

In Exercises 55–58, determine where the function is **(a)** differentiable, **(b)** continuous but not differentiable, and **(c)** neither continuous nor differentiable.

55. $f(x) = \sqrt[3]{(x - 1)^3}$

56. $g(x) = \sin(x^2 + 1)$

57. $f(x) = \begin{cases} \sqrt{x^2 + 3}, & -1 \le x < 1 \\ x + 1, & 1 \le x < 3 \end{cases}$

58. $g(x) = \begin{cases} \sin 2x, & -3 \le x < 0 \\ x^2 + 2x + 1, & 0 \le x \le 3 \end{cases}$

59. Simplify, Simplify Find the derivative of each function easily by simplifying the expression before differentiating.

(a) $y = (\sqrt{3 - \sin x})^2$ (b) $y = \ln(3e^{7x^2 - 13x + 5})$

(c) $s = \tan(\tan^{-1}(t^2 - 3t))$ (d) $s = \sqrt[3]{t^6} - 5(\sin(\sin^{-1} t))^6$

60. Simplify, Simplify Find the derivative of each function relatively easily by simplifying the expression before differentiating.

(a) $y = \ln\left(\dfrac{2x + 7}{3x + 2}\right)$ (b) $y = \dfrac{(x^2 - 1)^2}{(x^2 - 2x + 1)(x + 1))}$

(c) $s = \sin^2(\cos^{-1} t)$ (d) $s = \left(\dfrac{2\sqrt{t}}{\sqrt[3]{t}}\right)^5$

61. Hitting the Slopes A differentiable function f has the property that $f'(x) > 0$ for all x. The line $y = 3x - 2$ is tangent to the graph of f at the point where $x = 2$.

(a) Find the equation of the line normal to the graph of f at the point where $x = 2$.

(b) Find the equation of the line tangent to the graph of f^{-1} at the point where $y = 2$.

(c) Find the equation of the line tangent to the graph of $y = \dfrac{f(x)}{x}$ at the point where $x = 2$.

62. Hitting the Slopes Again A differentiable function g has the property that $g'(x) < 0$ for all x. The line $y = 5 - 2x$ is tangent to the graph of g at the point where $x = 1$.

(a) Find the equation of the line normal to the graph of g at the point where $x = 1$.

(b) Find the equation of the line tangent to the graph of g^{-1} at the point where $y = 1$.

(c) Find the equation of the line tangent to the graph of $y = g(x^2)$ at the point where $x = 1$.

63. Logarithmic Differentiation Use the technique of logarithmic differentiation to find $\dfrac{dy}{dx}$ when

$y = \dfrac{(x + 2)^5(2x - 3)^4}{(x + 17)^2}$. (See Example 7 of Section 4.4.)

64. Logarithmic Differentiation Use the technique of logarithmic differentiation to find $\dfrac{dy}{dx}$ when $y = (x^2 + 2)^{x+5}$. (See Example 7 of Section 4.4.)

65. Differential Equations Equations that involve derivatives are called *differential equations*. We will have much more to say about them in Chapter 7, but meanwhile try to use what you have learned in Chapters 3 and 4 to find at least one nonzero function that satisfies each of these differential equations.

(a) $f'(x) = x$ (b) $f'(x) = f(x)$ (c) $f'(x) = -f(x)$

(d) $f''(x) = f(x)$ (e) $f''(x) = -f(x)$

66. *Working with Numerical Values* Suppose that a function f and its first derivative have the following values at $x = 0$ and $x = 1$.

x	$f(x)$	$f'(x)$
0	9	-2
1	-3	1/5

Find the first derivative of the following combinations at the given value of x.

(a) $\sqrt{x}f(x)$, $x = 1$ (b) $\sqrt{f(x)}$, $x = 0$

(c) $f(\sqrt{x})$, $x = 1$ (d) $f(1 - 5\tan x)$, $x = 0$

(e) $\dfrac{f(x)}{2 + \cos x}$, $x = 0$ (f) $10\sin\left(\dfrac{\pi x}{2}\right)f^2(x)$, $x = 1$

67. *Working with Numerical Values* Suppose that functions f and g and their first derivatives have the following values at $x = -1$ and $x = 0$.

x	$f(x)$	$g(x)$	$f'(x)$	$g'(x)$
-1	0	-1	2	1
0	-1	-3	-2	4

Find the first derivative of the following combinations at the given value of x.

(a) $\dfrac{f(2x)}{x - 1}$, $x = 0$ (b) $f^2(x)g^3(x)$, $x = 0$

(c) $g(f(x))$, $x = -1$ (d) $f(g(x))$, $x = -1$

(e) $f(g(2x - 1))$, $x = 0$ (f) $g(x + f(x))$, $x = 0$

68. Find the value of dw/ds at $s = 0$ if $w = \sin(\sqrt{r} - 2)$ and $r = 8\sin(s + \pi/6)$.

69. Find the value of dr/dt at $t = 0$ if $r = (\theta^2 + 7)^{1/3}$ and $\theta^2 t + \theta = 1$.

70. *Particle Motion* The position at time $t \ge 0$ of a particle moving along the s-axis is

$$s(t) = 10\cos(t + \pi/4).$$

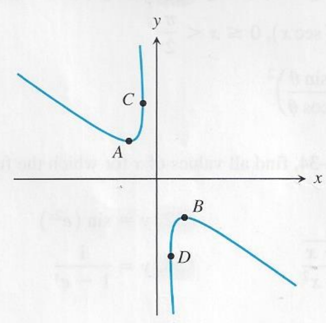
(a) Give parametric equations that can be used to simulate the motion of the particle.

(b) What is the particle's initial position ($t = 0$)?

(c) What points reached by the particle are farthest to the left and right of the origin?

(d) When does the particle first reach the origin? What are its velocity, speed, and acceleration then?

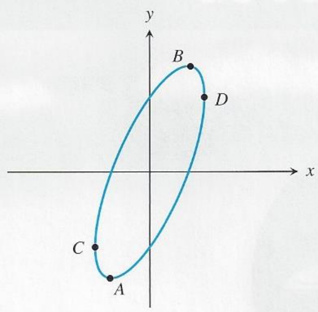
71. Implicit Hyperbola The hyperbola shown in the graph is defined implicitly by the equation $4x^2 + 8xy + y^2 + 3 = 0$.



(a) Find the coordinates of points A and B, where the tangent lines are horizontal.

(b) Find the coordinates of points C and D, where the tangent lines are vertical.

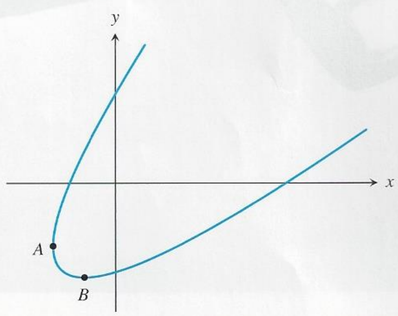
72. Implicit Ellipse The ellipse shown in the graph is defined implicitly by the equation $2x^2 - 2xy + y^2 - 4 = 0$.



(a) Find the coordinates of points A and B, where the tangent lines are horizontal.

(b) Find the coordinates of points C and D, where the tangent lines are vertical.

73. Implicit Parabola The parabola shown in the graph is defined implicitly by the equation $x^2 - 2xy + y^2 - 4x = 8$.



(a) Find the coordinates of point A, where the tangent line is vertical.

(b) Find the coordinates of point B, where the tangent line is horizontal.

74. Problem 73 Revisited Find the slope of the parabola defined implicitly by the equation $x^2 - 2xy + y^2 - 4x = 8$ at each of the points where it crosses the coordinate axes.

75. Slope of a Sinusoid Recall that the general equation of a sinusoid is $y = A \sin(Bx + C) + D$, where only the constants A and B affect the amplitude and period. What is the maximum possible slope for a sinusoid with amplitude 3 and period π?

76. Writing to Learn Write a formula that gives the maximum possible slope for a sinusoid with amplitude A and period p. Justify your answer.

77. Horizontal Tangents The graph of $y = \sin(x - \sin x)$ appears to have horizontal tangents at the x-axis. Does it?

78. *Spread of Measles* The spread of measles in a certain school is given by

$$P(t) = \frac{200}{1 + e^{5-t}},$$

where t is the number of days since the measles first appeared, and $P(t)$ is the total number of students who have caught the measles to date.

(a) Estimate the initial number of students infected with measles.

(b) About how many students in all will get the measles?

(c) When will the rate of spread of measles be greatest? What is this rate?

79. If $x^2 + 2xy + 2y^2 = 5$, find

(a) $\dfrac{dy}{dx}$ at the point $(1, 1)$;

(b) $\dfrac{d^2y}{dx^2}$ at the point $(1, 1)$.

80. If $x^2 - y^2 = 1$, find d^2y/dx^2 at the point $(2, \sqrt{3})$.

AP* Examination Preparation

81. A function f and its first and second derivatives are defined for all real numbers, and it is given that $f(0) = 2$, $f'(0) = 3$, and $f''(0) = -1$.

(a) Define a function g by $g(x) = e^{kx} + f(x)$, where k is a constant. Find $g'(0)$ and $g''(0)$ in terms of k. Show your work.

(b) Define a function h by $h(x) = \cos(bx)\, f(x)$, where b is a constant. Find $h'(x)$ and write an equation for the line tangent to the graph of h at $x = 0$.

82. Let $y = \dfrac{e^x + e^{-x}}{2}$.

(a) Find $\dfrac{dy}{dx}$.

(b) Find $\dfrac{d^2y}{dx^2}$.

(c) Find an equation of the line tangent to the curve at $x = 1$.

(d) Find an equation of the line normal to the curve at $x = 1$.

(e) Find any points where the tangent line is horizontal.

83. Let $f(x) = \ln(1 - x^2)$.

(a) State the domain of f.

(b) Find $f'(x)$.

(c) State the domain of f'.

(d) Prove that $f''(x) < 0$ for all x in the domain of f.