

Section 4.2 Exercises

In Exercises 1–8, find dy/dx .

1. $x^2y + xy^2 = 6$
2. $x^3 + y^3 = 18xy$
3. $y^2 = \frac{x-1}{x+1}$
4. $x^2 = \frac{x-y}{x+y}$
5. $x = \tan y$
6. $x = \sin y$
7. $x + \tan(xy) = 0$
8. $x + \sin y = xy$

In Exercises 9–12, find dy/dx and find the slope of the curve at the indicated point.

9. $x^2 + y^2 = 13$, $(-2, 3)$
10. $x^2 + y^2 = 9$, $(0, 3)$
11. $(x-1)^2 + (y-1)^2 = 13$, $(3, 4)$
12. $(x+2)^2 + (y+3)^2 = 25$, $(1, -7)$

In Exercises 13–16, find where the slope of the curve is defined.

13. $x^2y - xy^2 = 4$
14. $x = \cos y$
15. $x^3 + y^3 = xy$
16. $x^2 + 4xy + 4y^2 - 3x = 6$

In Exercises 17–26, find the lines that are (a) tangent and (b) normal to the curve at the given point.

17. $x^2 + xy - y^2 = 1$, $(2, 3)$
18. $x^2 + y^2 = 25$, $(3, -4)$
19. $x^2y^2 = 9$, $(-1, 3)$

20. $y^2 - 2x - 4y - 1 = 0$, $(-2, 1)$
21. $6x^2 + 3xy + 2y^2 + 17y - 6 = 0$, $(-1, 0)$
22. $x^2 - \sqrt{3}xy + 2y^2 = 5$, $(\sqrt{3}, 2)$
23. $2xy + \pi \sin y = 2\pi$, $(1, \pi/2)$
24. $x \sin 2y = y \cos 2x$, $(\pi/4, \pi/2)$
25. $y = 2 \sin(\pi x - y)$, $(1, 0)$
26. $x^2 \cos^2 y - \sin y = 0$, $(0, \pi)$

In Exercises 27–30, use implicit differentiation to find dy/dx and then d^2y/dx^2 .

27. $x^2 + y^2 = 1$
28. $x^{2/3} + y^{2/3} = 1$
29. $y^2 = x^2 + 2x$
30. $y^2 + 2y = 2x + 1$

In Exercises 31–42, find dy/dx .

31. $y = x^{9/4}$
32. $y = x^{-3/5}$
33. $y = \sqrt[3]{x}$
34. $y = \sqrt[4]{x}$
35. $y = (2x + 5)^{-1/2}$
36. $y = (1 - 6x)^{2/3}$
37. $y = x\sqrt{x^2 + 1}$
38. $y = \frac{x}{\sqrt{x^2 + 1}}$
39. $y = \sqrt{1 - \sqrt{x}}$
40. $y = 3(2x^{-1/2} + 1)^{-1/3}$
41. $y = 3(\csc x)^{3/2}$
42. $y = [\sin(x + 5)]^{5/4}$

43. Which of the following could be true if $f''(x) = x^{-1/3}$?

(a) $f(x) = \frac{3}{2}x^{2/3} - 3$ (b) $f(x) = \frac{9}{10}x^{5/3} - 7$
 (c) $f'''(x) = -\frac{1}{3}x^{-4/3}$ (d) $f'(x) = \frac{3}{2}x^{2/3} + 6$

44. Which of the following could be true if $g''(t) = 1/t^{3/4}$?

(a) $g'(t) = 4\sqrt[4]{t} - 4$ (b) $g'''(t) = -4/\sqrt[4]{t}$
 (c) $g(t) = t - 7 + (16/5)t^{5/4}$ (d) $g'(t) = (1/4)t^{1/4}$

45. **The Eight Curve** (a) Find the slopes of the figure-eight-shaped curve

$$y^4 = y^2 - x^2$$

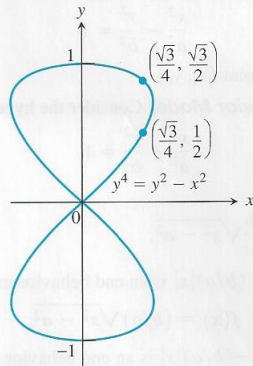
at the two points shown on the graph that follows.

- (b) Use parametric mode and the two pairs of parametric equations

$$x_1(t) = \sqrt{t^2 - t^4}, \quad y_1(t) = t,$$

$$x_2(t) = -\sqrt{t^2 - t^4}, \quad y_2(t) = t,$$

to graph the curve. Specify a window and a parameter interval.



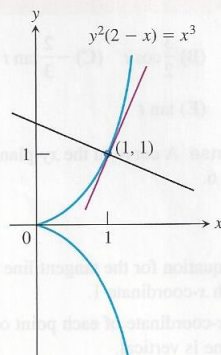
46. **The Cissoid of Diocles (dates from about 200 B.C.E.)**

- (a) Find equations for the tangent and normal to the cissoid of Diocles,

$$y^2(2 - x) = x^3,$$

at the point $(1, 1)$ as pictured below.

- (b) Explain how to reproduce the graph on a grapher.



47. (a) Confirm that $(-1, 1)$ is on the curve defined by

$$x^3y^2 = \cos(\pi y).$$

- (b) Use part (a) to find the slope of the line tangent to the curve at $(-1, 1)$.

48. Group Activity

- (a) Show that the relation

$$y^3 - xy = -1$$

cannot be a function of x by showing that there is more than one possible y value when $x = 2$.

- (b) On a small enough square with center $(2, 1)$, the part of the graph of the relation within the square will define a function $y = f(x)$. For this function, find $f'(2)$ and $f''(2)$.

49. Find the two points where the curve $x^2 + xy + y^2 = 7$ crosses the x -axis, and show that the tangents to the curve at these points are parallel. What is the common slope of these tangents?

50. Find points on the curve $x^2 + xy + y^2 = 7$ (a) where the tangent is parallel to the x -axis and (b) where the tangent is parallel to the y -axis. (In the latter case, dy/dx is not defined, but dx/dy is. What value does dx/dy have at these points?)

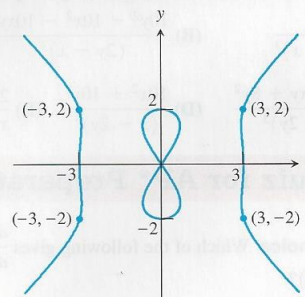
51. **Orthogonal Curves** Two curves are *orthogonal* at a point of intersection if their tangents at that point cross at right angles. Show that the curves $2x^2 + 3y^2 = 5$ and $y^2 = x^3$ are orthogonal at $(1, 1)$ and $(1, -1)$. Use parametric mode to draw the curves and to show the tangent lines.

52. The position of a body moving along a coordinate line at time t is $s = (4 + 6t)^{3/2}$, with s in meters and t in seconds. Find the body's velocity and acceleration when $t = 2$ sec.

53. The velocity of a falling body is $v = 8\sqrt{s - t} + 1$ feet per second at the instant t (sec) the body has fallen s feet from its starting point. Show that the body's acceleration is 32 ft/sec².

54. **The Devil's Curve (Gabriel Cramer [the Cramer of Cramer's Rule], 1750)** Find the slopes of the devil's curve $y^4 - 4y^2 = x^4 - 9x^2$ at the four indicated points.

$$y^4 - 4y^2 = x^4 - 9x^2$$



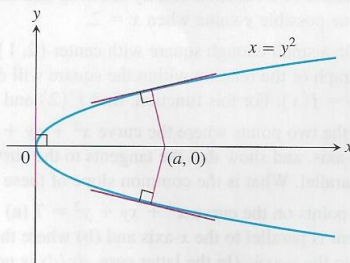
55. **The Folium of Descartes** (See Figure 4.7 on page 164.)

- (a) Find the slope of the folium of Descartes, $x^3 + y^3 - 9xy = 0$ at the points $(4, 2)$ and $(2, 4)$.

- (b) At what point other than the origin does the folium have a horizontal tangent?

- (c) Find the coordinates of point A in Figure 4.7, where the folium has a vertical tangent.

56. The line that is normal to the curve $x^2 + 2xy - 3y^2 = 0$ at $(1, 1)$ intersects the curve at what other point?
57. Find the normals to the curve $xy + 2x - y = 0$ that are parallel to the line $2x + y = 0$.
58. Show that if it is possible to draw three normals from the point $(a, 0)$ to the parabola $x = y^2$ shown here, then a must be greater than $1/2$. One of the normals is the x -axis. For what value of a are the other two normals perpendicular?



Standardized Test Questions

59. **True or False** The slope of $xy^2 + x = 1$ at $(1/2, 1)$ is 2. Justify your answer.
60. **True or False** The derivative of $y = \sqrt[3]{x}$ is $\frac{1}{3x^{2/3}}$. Justify your answer.
- In Exercises 61 and 62, use the curve $x^2 - xy + y^2 = 1$.
61. **Multiple Choice** Which of the following is equal to dy/dx ?
- (A) $\frac{y-2x}{2y-x}$ (B) $\frac{y+2x}{2y-x}$ (C) $\frac{2x}{x-2y}$
 (D) $\frac{2x+y}{x-2y}$ (E) $\frac{y+2x}{x}$
62. **Multiple Choice** Which of the following is equal to $\frac{d^2y}{dx^2}$?
- (A) $-\frac{6}{(2y-x)^3}$ (B) $\frac{10y^2 - 10x^2 - 10xy}{(2y-x)^3}$
 (C) $\frac{8x^2 - 4xy + 8y^2}{(x-2y)^3}$ (D) $\frac{10x^2 + 10y^2}{(x-2y)^3}$ (E) $\frac{2}{x}$

63. **Multiple Choice** Which of the following is equal to dy/dx if $y = x^{3/4}$?

(A) $\frac{3x^{1/3}}{4}$ (B) $\frac{4x^{1/4}}{3}$ (C) $\frac{3x^{1/4}}{4}$ (D) $\frac{4}{3x^{1/4}}$ (E) $\frac{3}{4x^{1/4}}$

64. **Multiple Choice** Which of the following is equal to the slope of the tangent to $y^2 - x^2 = 1$ at $(1, \sqrt{2})$?

(A) $-\frac{1}{\sqrt{2}}$ (B) $-\sqrt{2}$ (C) $\frac{1}{\sqrt{2}}$ (D) $\sqrt{2}$ (E) 0

Extending the Ideas

65. Finding Tangents

- (a) Show that the tangent to the ellipse

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

at the point (x_1, y_1) has equation

$$\frac{x_1x}{a^2} + \frac{y_1y}{b^2} = 1.$$

- (b) Find an equation for the tangent to the hyperbola

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

at the point (x_1, y_1) .

66. **End Behavior Model** Consider the hyperbola

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1.$$

Show that

(a) $y = \pm \frac{b}{a} \sqrt{x^2 - a^2}$.

- (b) $g(x) = (b/a)|x|$ is an end behavior model for

$$f(x) = (b/a)\sqrt{x^2 - a^2}.$$

- (c) $g(x) = -(b/a)|x|$ is an end behavior model for

$$f(x) = -(b/a)\sqrt{x^2 - a^2}.$$