Section 4.3 Exercises

In Exercises 1–8, find the derivative of y with respect to the appropriate variable.

1. $y = \cos^{-1}(x^2)$

2. $y = \cos^{-1}(1/x)$

3. $y = \sin^{-1}\sqrt{2t}$

4. $y = \sin^{-1}(1 - t)$

5. $y = \sin^{-1}\dfrac{3}{t^2}$

6. $y = s\sqrt{1 - s^2} + \cos^{-1} s$

7. $y = x\sin^{-1} x + \sqrt{1 - x^2}$

8. $y = \dfrac{1}{\sin^{-1}(2x)}$

In Exercises 9–12, a particle moves along the x-axis so that its position at any time $t \geq 0$ is given by $x(t)$. Find the velocity at the indicated value of t.

9. $x(t) = \sin^{-1}\left(\dfrac{t}{4}\right)$, $t = 3$

10. $x(t) = \sin^{-1}\left(\dfrac{\sqrt{t}}{4}\right)$, $t = 4$

11. $x(t) = \tan^{-1} t$, $t = 2$

12. $x(t) = \tan^{-1}(t^2)$, $t = 1$

In Exercises 13–22, find the derivative of y with respect to the appropriate variable.

13. $y = \sec^{-1}(2s + 1)$

14. $y = \sec^{-1} 5s$

15. $y = \csc^{-1}(x^2 + 1)$, $x > 0$

16. $y = \csc^{-1} x/2$

17. $y = \sec^{-1}\dfrac{1}{t}$, $0 < t < 1$

18. $y = \cot^{-1}\sqrt{t}$

19. $y = \cot^{-1}\sqrt{t - 1}$

20. $y = \sqrt{s^2 - 1} - \sec^{-1} s$

21. $y = \tan^{-1}\sqrt{x^2 - 1} + \csc^{-1} x$, $x > 1$,

22. $y = \cot^{-1}\dfrac{1}{x} - \tan^{-1} x$

In Exercises 23–26, find an equation for the tangent to the graph of y at the indicated point. Write your answer in the form $y = ax + b$ with a and b correct to the nearest thousandth.

23. $y = \sec^{-1} x$, $x = 2$

24. $y = \tan^{-1} x$, $x = 2$

25. $y = \sin^{-1}\left(\dfrac{x}{4}\right)$, $x = 3$

26. $y = \tan^{-1}(x^2)$, $x = 1$

27. **(a)** Find an equation for the line tangent to the graph of $y = \tan x$ at the point $(\pi/4, 1)$.

(b) Find an equation for the line tangent to the graph of $y = \tan^{-1} x$ at the point $(1, \pi/4)$.

28. Let $f(x) = x^5 + 2x^3 + x - 1$.

(a) Find $f(1)$ and $f'(1)$.

(b) Find $f^{-1}(3)$ and $(f^{-1})'(3)$.

29. Let $f(x) = \cos x + 3x$.

(a) Show that f has a differentiable inverse.

(b) Find $f(0)$ and $f'(0)$.

(c) Find $f^{-1}(1)$ and $(f^{-1})'(1)$.

30. Group Activity Graph the function $f(x) = \sin^{-1}(\sin x)$ in the viewing window $[-2\pi, 2\pi]$ by $[-4, 4]$. Then answer the following questions:

(a) What is the domain of f?

(b) What is the range of f?

(c) At which points is f not differentiable?

(d) Sketch a graph of $y = f'(x)$ without using NDER or computing the derivative.

(e) Find $f'(x)$ algebraically. Can you reconcile your answer with the graph in part (d)?

31. Group Activity A particle moves along the x-axis so that its position at any time $t \geq 0$ is given by $x = \arctan t$.

(a) Prove that the particle is always moving to the right.

(b) Prove that the particle is always decelerating.

(c) What is the limiting position of the particle as t approaches infinity?

In Exercises 32–34, use the inverse function–inverse cofunction identities to derive the formula for the derivative of the function.

32. arccosine

33. arccotangent

34. arccosecant

Standardized Test Questions

You may use a graphing calculator to solve the following problems.

35. True or False The domain of $y = \sin^{-1} x$ is $-1 \le x \le 1$. Justify your answer.

36. True or False The domain of $y = \tan^{-1} x$ is $-1 \le x \le 1$. Justify your answer.

37. Multiple Choice Which of the following is $\dfrac{d}{dx}\sin^{-1}\left(\dfrac{x}{2}\right)$?

(A) $-\dfrac{2}{\sqrt{4-x^2}}$ (B) $-\dfrac{1}{\sqrt{4-x^2}}$ (C) $\dfrac{2}{4+x^2}$

(D) $\dfrac{2}{\sqrt{4-x^2}}$ (E) $\dfrac{1}{\sqrt{4-x^2}}$

38. Multiple Choice Which of the following is $\dfrac{d}{dx}\tan^{-1}(3x)$?

(A) $-\dfrac{3}{1+9x^2}$ (B) $-\dfrac{1}{1+9x^2}$ (C) $\dfrac{1}{1+9x^2}$

(D) $\dfrac{3}{1+9x^2}$ (E) $\dfrac{3}{\sqrt{1-9x^2}}$

39. Multiple Choice Which of the following is $\dfrac{d}{dx}\sec^{-1}(x^2)$?

(A) $\dfrac{2}{x\sqrt{x^4-1}}$ (B) $\dfrac{2}{x\sqrt{x^2-1}}$ (C) $\dfrac{2}{x\sqrt{1-x^4}}$

(D) $\dfrac{2}{x\sqrt{1-x^2}}$ (E) $\dfrac{2x}{\sqrt{1-x^4}}$

40. Multiple Choice Which of the following is the slope of the tangent line to $y = \tan^{-1}(2x)$ at $x = 1$?

(A) $-2/5$ (B) $1/5$ (C) $2/5$ (D) $5/2$ (E) 5

Explorations

In Exercises 41–46, find (**a**) the right end behavior model, (**b**) the left end behavior model, and (**c**) any horizontal tangents for the function if they exist.

41. $y = \tan^{-1} x$ **42.** $y = \cot^{-1} x$

43. $y = \sec^{-1} x$ **44.** $y = \csc^{-1} x$

45. $y = \sin^{-1} x$ **46.** $y = \cos^{-1} x$

Extending the Ideas

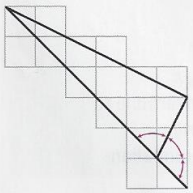
47. *Identities* Confirm the following identities for $x > 0$.

(**a**) $\cos^{-1} x + \sin^{-1} x = \pi/2$

(**b**) $\tan^{-1} x + \cot^{-1} x = \pi/2$

(**c**) $\sec^{-1} x + \csc^{-1} x = \pi/2$

48. *Proof Without Words* The figure gives a proof without words that $\tan^{-1} 1 + \tan^{-1} 2 + \tan^{-1} 3 = \pi$. Explain what is going on.



49. *(Continuation of Exercise 48)* Here is a way to construct $\tan^{-1} 1$, $\tan^{-1} 2$, and $\tan^{-1} 3$ by folding a square of paper. Try it and explain what is going on.