

## Section 4.4 Exercises

In Exercises 1–28, find  $dy/dx$ .

1.  $y = 2e^x$

2.  $y = e^{2x}$

3.  $y = e^{-x}$

4.  $y = e^{-5x}$

5.  $y = e^{2x/3}$

6.  $y = e^{-x/4}$

7.  $y = xe^2 - e^x$

8.  $y = x^2e^x - xe^x$

9.  $y = e^{\sqrt{x}}$

10.  $y = e^{(x^2)}$

11.  $y = 8^x$

12.  $y = 9^{-x}$

13.  $y = 3^{\csc x}$

14.  $y = 3^{\cot x}$

15.  $y = \ln(x^2)$

16.  $y = (\ln x)^2$

17.  $y = \ln(1/x)$

18.  $y = \ln(10/x)$

19.  $y = \ln(\ln x)$

20.  $y = x \ln x - x$

21.  $y = \log_4 x^2$

22.  $y = \log_5 \sqrt{x}$

23.  $y = \log_2(1/x)$

24.  $y = 1/\log_2 x$

25.  $y = \ln 2 \cdot \log_2 x$

26.  $y = \log_3(1 + x \ln 3)$

27.  $y = \log_{10} e^x$

28.  $y = \ln 10^x$

29. At what point on the graph of  $y = 3^x + 1$  is the tangent line parallel to the line  $y = 5x - 1$ ?

30. At what point on the graph of  $y = 2e^x - 1$  is the tangent line perpendicular to the line  $y = -3x + 2$ ?

31. A line with slope  $m$  passes through the origin and is tangent to  $y = \ln(2x)$ . What is the value of  $m$ ?

32. A line with slope  $m$  passes through the origin and is tangent to  $y = \ln(x/3)$ . What is the value of  $m$ ?

In Exercises 33–36, find  $dy/dx$ .

33.  $y = x^\pi$

34.  $y = x^{1+\sqrt{2}}$

35.  $y = x^{-\sqrt{2}}$

36.  $y = x^{1-e}$

In Exercises 37–42, find  $f'(x)$  and state the domain of  $f'$ .

37.  $f(x) = \ln(x + 2)$

38.  $f(x) = \ln(2x + 2)$

39.  $f(x) = \ln(2 - \cos x)$

40.  $f(x) = \ln(x^2 + 1)$

41.  $f(x) = \log_2(3x + 1)$

42.  $f(x) = \log_{10}\sqrt{x + 1}$

**Group Activity** In Exercises 43–48, use the technique of logarithmic differentiation to find  $dy/dx$ .

43.  $y = (\sin x)^x, \quad 0 < x < \pi/2$

44.  $y = x^{\tan x}, \quad x > 0$

45.  $y = \sqrt[5]{\frac{(x-3)^4(x^2+1)}{(2x+5)^3}}$

46.  $y = \frac{x\sqrt{x^2+1}}{(x+1)^{2/3}}$

47.  $y = x^{\ln x}$

48.  $y = x^{(1/\ln x)}$

49. Find an equation for a line that is tangent to the graph of  $y = e^x$  and goes through the origin.

50. Find an equation for a line that is normal to the graph of  $y = xe^x$  and goes through the origin.

51. **Spread of a Rumor** The spread of a rumor in a certain school is modeled by the equation

$$P(t) = \frac{300}{1 + 2^{4-t}}$$

where  $P(t)$  is the total number of students who have heard the rumor  $t$  days after the rumor first started to spread.

- Estimate the initial number of students who first heard the rumor.
- How fast is the rumor spreading after 4 days?
- When will the rumor spread at its maximum rate? What is that rate?

52. **Spread of Flu** The spread of flu in a certain school is modeled by the equation

$$P(t) = \frac{200}{1 + e^{5-t}}$$

where  $P(t)$  is the total number of students infected  $t$  days after the flu first started to spread.

- Estimate the initial number of students infected with this flu.
- How fast is the flu spreading after 4 days?
- When will the flu spread at its maximum rate? What is that rate?

53. **Radioactive Decay** The amount  $A$  (in grams) of radioactive plutonium remaining in a 20-gram sample after  $t$  days is given by the formula

$$A = 20 \cdot (1/2)^{t/140}$$

At what rate is the plutonium decaying when  $t = 2$  days? Answer in appropriate units.

54. For any positive constant  $k$ , the derivative of  $\ln(kx)$  is  $1/x$ . Prove this fact

- by using the Chain Rule.
- by using a property of logarithms and differentiating.

55. Let  $f(x) = 2^x$ .

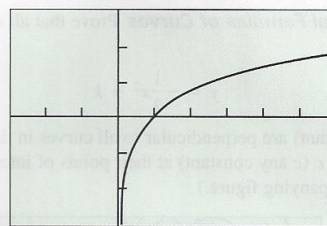
- Find  $f'(0)$ .
- Use the definition of the derivative to write  $f'(0)$  as a limit.
- Deduce the exact value of

$$\lim_{h \rightarrow 0} \frac{2^h - 1}{h}$$

(d) What is the exact value of

$$\lim_{h \rightarrow 0} \frac{7^h - 1}{h}?$$

56. **Writing to Learn** The graph of  $y = \ln x$  looks as though it might be approaching a horizontal asymptote. Write an argument based on the graph of  $y = e^x$  to explain why it does not.



$[-3, 6]$  by  $[-3, 3]$

## Standardized Test Questions

57. **True or False** The derivative of  $y = 2^x$  is  $2^x$ . Justify your answer.

58. **True or False** The derivative of  $y = e^{2x}$  is  $2(\ln 2)e^{2x}$ . Justify your answer.

59. **Multiple Choice** If a flu is spreading at the rate of

$$P(t) = \frac{150}{1 + e^{4-t}},$$

which of the following is the initial number of persons infected?

- (A) 1 (B) 3 (C) 7 (D) 8 (E) 75

60. **Multiple Choice** Which of the following is the domain of  $f'(x)$  if  $f(x) = \log_2(x + 3)$ ?

- (A)  $x < -3$  (B)  $x \leq 3$  (C)  $x \neq -3$   
(D)  $x > -3$  (E)  $x \geq -3$

61. **Multiple Choice** Which of the following gives  $dy/dx$  if  $y = \log_{10}(2x - 3)$ ?

- (A)  $\frac{2}{(2x-3)\ln 10}$  (B)  $\frac{2}{2x-3}$  (C)  $\frac{1}{(2x-3)\ln 10}$   
(D)  $\frac{1}{2x-3}$  (E)  $\frac{1}{2x}$

62. **Multiple Choice** Which of the following gives the slope of the tangent line to the graph of  $y = 2^{1-x}$  at  $x = 2$ ?

- (A)  $-\frac{1}{2}$  (B)  $\frac{1}{2}$  (C)  $-2$  (D)  $2$  (E)  $-\frac{\ln 2}{2}$

## Explorations

63. Let  $y_1 = a^x$ ,  $y_2 = \text{NDER}(y_1, x)$ ,  $y_3 = y_2/y_1$ , and  $y_4 = e^{y_3}$ .

- Describe the graph of  $y_4$  for  $a = 2, 3, 4, 5$ . Generalize your description to an arbitrary  $a > 1$ .
- Describe the graph of  $y_3$  for  $a = 2, 3, 4, 5$ . Compare a table of values for  $y_3$  for  $a = 2, 3, 4, 5$  with  $\ln a$ . Generalize your description to an arbitrary  $a > 1$ .
- Explain how parts (a) and (b) support the statement

$$\frac{d}{dx} a^x = a^x \quad \text{if and only if} \quad a = e.$$

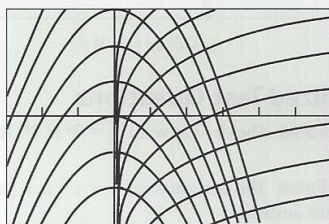
- Show algebraically that  $y_1 = y_2$  if and only if  $a = e$ .

## Extending the Ideas

64. **Orthogonal Families of Curves** Prove that all curves in the family

$$y = -\frac{1}{2}x^2 + k$$

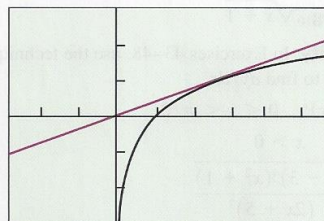
( $k$  any constant) are perpendicular to all curves in the family  $y = \ln x + c$  ( $c$  any constant) at their points of intersection. (See accompanying figure.)



$[-3, 6]$  by  $[-3, 3]$

65. **Which Is Bigger,  $\pi^e$  or  $e^\pi$ ?** Calculators have taken some of the mystery out of this once-challenging question. (Go ahead and check; you will see that it is a surprisingly close call.) You can answer the question without a calculator, though, by using the result from Example 3 of this section.

Recall from that example that the line through the origin tangent to the graph of  $y = \ln x$  has slope  $1/e$ .



$[-3, 6]$  by  $[-3, 3]$

- Find an equation for this tangent line.
- Give an argument based on the graphs of  $y = \ln x$  and the tangent line to explain why  $\ln x < x/e$  for all positive  $x \neq e$ .
- Show that  $\ln(x^e) < x$  for all positive  $x \neq e$ .
- Conclude that  $x^e < e^x$  for all positive  $x \neq e$ .
- So which is bigger,  $\pi^e$  or  $e^\pi$ ?