Hw #5

4.1. Chain Rule

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(1) $y = \sin(3x+1)$, $y' = \cos(3x+1)\cdot 3 = 3\cos(3x+1)$

(3) $y = \cos(\sqrt{3}x)$, $y' = -\sin(\sqrt{3}x)\cdot\sqrt{3} = -\sqrt{3}\sin(\sqrt{3}x)$

(5) $y = g^2$, $g = \dfrac{\sin x}{1+\cos x} = \dfrac{u}{v}$, $\dfrac{dg}{dx} = \dfrac{u'v - uv'}{v^2} = \dfrac{\cos x(1+\cos x) - \sin x\cdot -\sin x}{(1+\cos x)^2} =$

$= \dfrac{\cos x + \cos^2 x + \sin^2 x}{(1+\cos x)^2} = \dfrac{1+\cos x}{(1+\cos x)^2} = \dfrac{1}{1+\cos x}$, $\dfrac{dy}{dx} = \dfrac{dy}{dg}\dfrac{dg}{dx} =$

$= 2g\cdot\dfrac{1}{(1+\cos x)} = \dfrac{2\sin x}{(1+\cos x)}\cdot\dfrac{1}{(1+\cos x)} = \dfrac{2\sin x}{(1+\cos x)^2}$

(7) $y = \cos(\sin x)$, $y' = -\sin(\sin x)\cdot\cos x$

(13) $y = \left[x + x^{1/2}\right]^{-2}$, $y' = -2\left[x + x^{1/2}\right]^{-3}\cdot\left(1 + \frac{1}{2}x^{-1/2}\right) =$

$= \dfrac{-2}{(x+\sqrt{x})^3}\cdot\left(\dfrac{2\sqrt{x}}{2\sqrt{x}} + \dfrac{1}{2\sqrt{x}}\right) = \dfrac{-2(1+2\sqrt{x})}{2\sqrt{x}(x+\sqrt{x})^3}$

(16) $y = x^3(2x-5)^4$, $y' = [x^3]'(2x-5)^4 + x^3\left[(2x-5)^4\right]' =$

$= 3x^2(2x-5)^4 + x^3\cdot 4(2x-5)^3\cdot 2 = 3x^2(2x-5)^4 + 8x^3(2x-5)^3$

(23) $y = (1+\cos^2 7x)^3$, $y' = 3(1+\cos^2 7x)^2\cdot 2\cos 7x\cdot -\sin 7x\cdot 7 =$

$= -42\sin 7x\cos 7x(1+\cos^2 7x)^2$

(24) $y = (\tan 5x)^{1/2}$, $y' = \frac{1}{2}(\tan 5x)^{-1/2}\cdot\sec^2 5x\cdot 5 = \dfrac{5\sec^2 5x}{2\sqrt{\tan 5x}}$

(51) $s = \cos\theta$ $\quad \dfrac{ds}{dt} = \dfrac{ds}{d\theta}\cdot\dfrac{d\theta}{dt} = -\sin\theta\cdot\dfrac{d\theta}{dt} = -\sin\left(\dfrac{3\pi}{2}\right)\cdot 5 = -1\cdot 5 = 5$

(67) $T = 2\pi\sqrt{\dfrac{L}{g}}$ $\quad \dfrac{dL}{du} = kL$, $\dfrac{dT}{du} = \dfrac{dT}{dL}\dfrac{dL}{du} = \dfrac{2\pi}{\sqrt{g}}\cdot\frac{1}{2}L^{-1/2}\cdot kL =$

$= \dfrac{\pi}{\sqrt{g}}\cdot k\sqrt{L} = k\cdot\pi\sqrt{\dfrac{L}{g}} = k\cdot\dfrac{T}{2} = \frac{1}{2}kT$

4.1. Numerical Values

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(56) $\frac{d}{dx}\left[f\cdot g\right] = f'(3)g(3) + f(3)g'(3) = 2\pi\cdot -4 + 3\cdot 5 = 15 - 8\pi$

(c) (d) $\frac{d}{dx}\left[\frac{f}{g}\right] = \frac{f'(2)g(2) - f(2)g'(2)}{[g(2)]^2} = \frac{\frac{1}{3}\cdot 2 - 8\cdot -3}{2^2} = \frac{\frac{2}{3}+24}{4} = \frac{\frac{74}{3}}{4} = \frac{74}{12} = \frac{37}{6}$

(e) $\frac{d}{dx}\left[f(g(x))\right] = \left.\frac{df}{dg}\right|_{\substack{x=2\\g=2}} \cdot \left.\frac{dg}{dx}\right|_{x=2} = f'(2)g'(2) = \frac{1}{3}\cdot -3 = -1$

(58) (d) $\frac{d}{dx}\left[f(g(x))\right] = \left.\frac{df}{dg}\right|_{\substack{x=0\\g=1}} \cdot \left.\frac{dg}{dx}\right|_{x=0} = f'(1)g'(0) = -\frac{1}{3}\cdot\frac{1}{3} = -\frac{1}{9}$

(e) $\frac{d}{dx}\left[g(f(x))\right] = \left.\frac{dg}{df}\right|_{\substack{x=0\\f=1}} \cdot \left.\frac{df}{dx}\right|_{x=0} = g'(1)\cdot f'(0) = -\frac{8}{3}\cdot 5 = -\frac{40}{3}$

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4.2. Implicit Differentiation

(1) $x^2y + xy^2 = 6$, $\frac{dx^2}{dx}y + x^2\frac{dy}{dx} + \frac{dx}{dx}y^2 + x\frac{dy^2}{dx} = 0$, $2xy + x^2\frac{dy}{dx} + y^2 + x\frac{dy^2}{dy}\frac{dy}{dx} = 0$,

$2xy + x^2\frac{dy}{dx} + y^2 + x\cdot 2y\frac{dy}{dx} = 0$, $2xy + y^2 + (x^2 + 2xy)\frac{dy}{dx} = 0$

$$\frac{dy}{dx} = -\frac{2xy + y^2}{2xy + x^2}$$

(4) $x^2 = \frac{x-y}{x+y} = \frac{u}{v}$, $2x = \frac{d}{dx}\left[\frac{u}{v}\right] = \frac{u'v - uv'}{v^2} = \frac{\left(1 - \frac{dy}{dx}\right)(x+y) - (x-y)\left(1 + \frac{dy}{dx}\right)}{(x+y)^2}$

$$\cancel{2x} = \frac{\left(x + y - x\frac{dy}{dx} - y\frac{dy}{dx}\right) - \left(x + x\frac{dy}{dx} - y - y\frac{dy}{dx}\right)}{(x+y)^2} =$$

$$= \frac{\cancel{x} + y - x\frac{dy}{dx} - \cancel{y\frac{dy}{dx}} - \cancel{x} - x\frac{dy}{dx} + y + \cancel{y\frac{dy}{dx}}}{(x+y)^2} = \frac{2y - 2x\frac{dy}{dx}}{(x+y)^2} = \frac{\cancel{2}\left(y - x\frac{dy}{dx}\right)}{(x+y)^2}$$

$x(x+y)^2 = y - x\frac{dy}{dx}$, $x(x+y)^2 - y = -x\frac{dy}{dx}$, $\frac{dy}{dx} = \frac{y - x(x+y)^2}{x}$

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(8) $x+\sin y = xy$, $1+\frac{d\sin y}{dx} = 1\cdot y + x\frac{dy}{dx}$, $1+\frac{d\sin y}{dy}\frac{dy}{dx} = y + x\frac{dy}{dx}$, $1+\cos y\frac{dy}{dx} = y + x\frac{dy}{dx}$,

$1-y = x\frac{dy}{dx} - \cos y\frac{dy}{dx} = (x-\cos y)\frac{dy}{dx}$, $\frac{dy}{dx} = \frac{1-y}{x-\cos y}$ ◀

(21)

(a) $6x^2+3xy+2y^2+17y-6=0$

$12x + 3\left(1\cdot y + x\frac{dy}{dx}\right) + 2\frac{dy^2}{dx} + 17\frac{dy}{dx} = 0$

$12x + 3\left(y + x\frac{dy}{dx}\right) + 2\frac{dy^2}{dy}\frac{dy}{dx} + 17\frac{dy}{dx} = 0$, $12x+3y+3x\frac{dy}{dx} + 4y\frac{dy}{dx} + 17\frac{dy}{dx} = 0$

$12x+3y+(3x+4y+17)\frac{dy}{dx} = 0$, $\frac{dy}{dx} = -\frac{12x+3y}{3x+4y+17}$, $\left.\frac{dy}{dx}\right|_{(-1,0)} = -\frac{12(-1)+3(0)}{3(-1)+4(0)+17} = \frac{6}{7}$

$y = \frac{6}{7}x + b$, $0 = \frac{6}{7}(-1)+b$, $b = \frac{6}{7}$ $\Rightarrow$ $y = \frac{6}{7}x + \frac{6}{7}$ ◀

(65)

(a) $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, $b^2x^2 + a^2y^2 = a^2b^2$, $b^2\cdot 2x + a^2\frac{dy^2}{dx} = 0$,

$b^2\cdot 2x + a^2\frac{dy^2}{dy}\frac{dy}{dx} = 0$, $b^2\cdot\cancel{2}x + a^2\cdot\cancel{2}y\frac{dy}{dx} = 0$, $\frac{dy}{dx} = -\frac{b^2x}{a^2y}$

$\left.\frac{dy}{dx}\right|_{(x_1,y_1)} = -\frac{b^2x_1}{a^2y_1}$, point-slope $y-y_1 = -\frac{b^2x_1}{a^2y_1}(x-x_1)$,

$\frac{(x-x_1)}{a^2y_1} + \frac{(y-y_1)}{b^2x_1} = 0$ $\frac{x_1(x-x_1)}{a^2} + \frac{y_1(y-y_1)}{b^2} = 0$, $\frac{xx_1}{a^2} - \frac{x_1^2}{a^2} + \frac{yy_1}{b^2} - \frac{y_1^2}{b^2} = 0$

$\frac{x_1x}{a^2} + \frac{y_1y}{b^2} = \frac{x_1^2}{a^2} + \frac{y_1^2}{b^2} = 1$ ◀

pg. 189 4.2. Implicit Differentiation – II

(71) $4x^2+8xy+y^2+3=0$, $4\cdot 2x + 8\left(1\cdot y + x\frac{dy}{dx}\right) + 2y\frac{dy}{dx} = 0$

$4x+4y+4x\frac{dy}{dx} + y\frac{dy}{dx} = 0$, $4x+4y+(4x+y)\frac{dy}{dx} = 0$, $\frac{dy}{dx} = -\frac{4x+4y}{4x+y}$

(a) $\frac{dy}{dx} = 0$, $4x+4y=0$, $x+y=0$, $x=-y$, $4(-y)^2+8(-y)y+y^2+3=0$,

$4y^2-8y^2+y^2+3=0$, $-3y^2+3=0$, $y^2=1$, $y=\pm 1$, $x=\mp 1$

$(-1,1)$ A ◀ $(1,-1)$ B ◀

(b) $\frac{dy}{dx} = \pm\infty$, $4x+y=0$, $y=-4x$, $4x^2+8x(-4x)+(-4x)^2+3=0$,

$4x^2-32x^2+16x^2+3=0$, $-12x^2+3=0$, $4x^2=1$, $x^2=\frac{1}{4}$, $x=\pm\frac{1}{2}$,

$y=\mp 2$, $(-\frac{1}{2},2)$ & $(\frac{1}{2},-2)$ ◄

Supplemental:

(1)

(a) $x^2y - xy^2 = 4$, $xy^2 - x^2y + 4 = 0$, $y = \frac{x^2 \pm \sqrt{x^4 - 4x(4)}}{2x} = \frac{x^2 \pm \sqrt{x^4-16x}}{2x}$ ◄

(c) V.A. at $x=0$ ◄ For $x \to \pm\infty$, $y \to \frac{x^2 \pm \sqrt{x^4}}{2x} = \frac{x^2 \pm x^2}{2x}$, $y=x$ O.A. ◄,

$y=0$ H.A. ◄

(d) $2x \cdot y + x^2\frac{dy}{dx} - \left(1\cdot y^2 + x\frac{dy^2}{dx}\right) = 0$, $2xy + x^2\frac{dy}{dx} - y^2 - 2xy\frac{dy}{dx} = 0$,

$2xy - y^2 + (x^2-2xy)\frac{dy}{dx} = 0$ $\quad \frac{dy}{dx} = \frac{y^2-2xy}{x^2-2xy}$ ◄

(e) $\frac{dy}{dx} = 0$, $y^2 - 2xy = 0$, $y(y-2x)=0$, $y=0$ (which is the H.A.),

$y=2x$, $x^2(2x) - x(2x)^2 = 4$, $2x^3 - 4x^3 = 4$, $-2x^3 = 4$, $x = -\sqrt[3]{2}$,

$y = -2\sqrt[3]{2}$, $(-\sqrt[3]{2}, -2\sqrt[3]{2}) \approx (-1.25\,99, -2.5198)$ ◄

(f) $\frac{dy}{dx} = \pm\infty$, $x^2 - 2xy = 0$, $x(x-2y)=0$, $x=0$ (which is the V.A.),

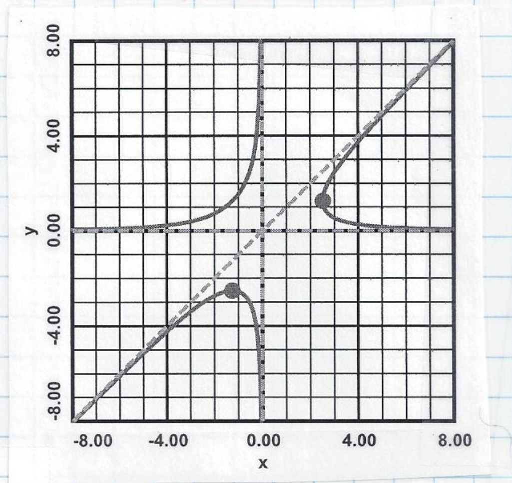
$x=2y$, $(2y)^2y - (2y)y^2 = 4$, $4y^3 - 2y^3 = 4$, $2y^3=4$, $y^3=2$, $y=\sqrt[3]{2}$,

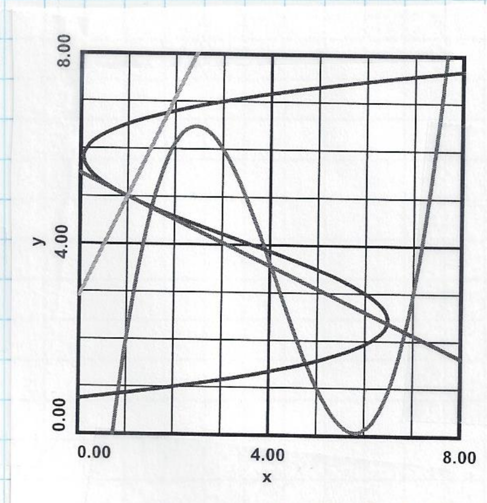
$x = 2y = 2\sqrt[3]{2}$, $(2\sqrt[3]{2}, \sqrt[3]{2}) = (2.5198, 1.2599)$

(g) $x \in (-\infty,0) \cup (2\cdot\sqrt[3]{2}, \infty)$ or $x \in (-\infty,0) \cup (2.5198, \infty)$ ◄

$y \in (-\infty, -2\cdot\sqrt[3]{2}) \cup (0,\infty)$ or $y \in (-\infty, -2.5198) \cup (0,\infty)$ ◄

(b)





(a)

4.3. Inverse Function Theorem

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(29) $f(x) = \cos x + 3x$

(b) $f'(x) = -\sin x + 3$, $f(0) = 1$, $f'(0) = 3$, $(0,1) \to (1,0)$

(c) $f^{-1}(1) = 0$, $\left.\dfrac{df^{-1}}{dx}\right|_{(1,0)} = \dfrac{1}{f'(0)} = \dfrac{1}{3}$

supplemental:

(2) $f(x) = \frac{1}{3}x^3 - \frac{33}{8}x^2 + \frac{85}{6}x - \frac{67}{8}$, $f'(x) = x^2 - \frac{33}{4}x + \frac{85}{6}$

(b) $f(5) = 1$, $f'(5) = -\frac{25}{12}$, $\left.\dfrac{df^{-1}}{dx}\right|_{(1,5)} = -\frac{12}{25}$, $y = -\frac{12}{25}x + b$,

$5 = -\frac{12}{25}(1) + b$, $b = \frac{137}{25}$, $y = -\frac{12}{25}x + \frac{137}{25}$

(c) $y = \frac{25}{12}x + b$, $5 = \frac{25}{12}(1) + b$, $b = \frac{35}{12}$, $y = \frac{25}{12}x + \frac{35}{12}$