Quiz #5 Study Guide

(1) $f'(x) = -\csc(3x^2+5x+7)\cot(3x^2+5x+7)\cdot(6x+5) =$
$= -(6x+5)\csc(3x^2+5x+7)\cot(3x^2+5x+7)$

(2) $f'(x) = -\sin(\cot 4x)\cdot -\csc^2 4x \cdot 4 = 4\sin(\cot 4x)\csc^2 4x$

(3) $f(x) = g^2$, $g = \frac{\sin x}{1+\cos x} = \frac{u}{v}$ $\frac{dg}{dx} = \frac{u'v-uv'}{v^2} = \frac{\cos x(1+\cos x)-\sin x\cdot -\sin x}{(1+\cos x)^2} =$

$= \frac{\cos x+\cos^2 x+\sin^2 x}{(1+\cos x)^2} = \frac{1+\cos x}{(1+\cos x)^2} = \frac{1}{1+\cos x}$, $\frac{df}{dx} = \frac{df}{dg}\cdot\frac{dg}{dx} =$

$= 2\left(\frac{\sin x}{1+\cos x}\right)\cdot\frac{1}{1+\cos x} = \frac{2\sin x}{(1+\cos x)^2}$

(4)

(a) $\frac{d}{dx}(f\cdot g) = f'(3)g(3)+f(3)g'(3) = 4\cdot 6+5\cdot 7 = 59$

(b) $\frac{d}{dx}\left(\frac{f}{g}\right) = \frac{f'(3)g(3)-f(3)g'(3)}{[g(3)]^2} = \frac{4\cdot 6-5\cdot 7}{6^2} = \frac{-11}{36}$

(c) $\frac{d}{dx}f(g(x)) = \left.\frac{df}{dg}\right|_{\substack{x=3\\g=6}}\cdot\left.\frac{dg}{dx}\right|_{x=3} = f'(6)\cdot g'(3) = 12\cdot 7 = 84$

(d) $\frac{d}{dx}g(f(x)) = \left.\frac{dg}{df}\right|_{\substack{x=3\\f=5}}\cdot\left.\frac{df}{dx}\right|_{x=3} = g'(5)\cdot f'(3) = 9\cdot 4 = 36$

(5) $\frac{dy}{dx}\tan xy + y\sec^2 xy\cdot\frac{d}{dx}(xy) = 0$, $\frac{dy}{dx}\tan xy + y\sec^2 xy\left(1\cdot y + x\frac{dy}{dx}\right) = 0$,

$\frac{dy}{dx}\tan xy + y^2\sec^2 xy + xy\sec^2 xy\frac{dy}{dx} = 0$

$y^2\sec^2 xy + \left(\tan xy + xy\sec^2 xy\right)\frac{dy}{dx} = 0$, $\frac{dy}{dx} = -\frac{y^2\sec^2 xy}{\tan xy + xy\sec^2 xy}$

(6) $4x^3y + x^4\frac{dy}{dx} + 2\left(1\cdot y^4 + x\cdot 4y^3\frac{dy}{dx}\right) = 0$, $4x^3y + x^4\frac{dy}{dx} + 2y^4 + 8xy^3\frac{dy}{dx} = 0$,

$4x^3y + 2y^4 + \left(x^4+8xy^3\right)\frac{dy}{dx} = 0$, $\frac{dy}{dx} = -\frac{4x^3y+2y^4}{8xy^3+x^4}$

$\left.\frac{dy}{dx}\right|_{(2,3)} = -\frac{4\cdot 2^3\cdot 3+2\cdot 3^4}{8\cdot 2\cdot 3^3+2^4} = -\frac{258}{448} = -\frac{129}{224}$, $y = -\frac{129}{224}x+b$,

$3 = -\frac{129}{224}(2)+b$, $b = \frac{465}{112}$, $y = -\frac{129}{224}x+\frac{465}{112}$

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(7) $2x + 2 \cdot 2y \frac{dy}{dx} - 4\left(1 \cdot y + x\frac{dy}{dx}\right) = 0$, $2x + 4y\frac{dy}{dx} - 4y - 4x\frac{dy}{dx} = 0$,

$x + 2y\frac{dy}{dx} - 2y - 2x\frac{dy}{dx} = 0$, $x - 2y + (2y - 2x)\frac{dy}{dx} = 0$, $\frac{dy}{dx} = \frac{2y - x}{2y - 2x}$

(a) $\frac{dy}{dx} = 0 \Rightarrow 2y - x = 0$, $x = 2y$, $(2y)^2 + 2y^2 - 4(2y)y = -7$, $4y^2 + 2y^2 - 8y^2 = -7$,

$-2y^2 = -7$, $2y^2 = 7$, $y = \pm 1.8708$, $x = \pm 3.7417$

$(-3.7417, -1.8708)$ ← $(3.7417, 1.8708)$ ←

(b) $\frac{dy}{dx} = \pm\infty \Rightarrow 2y - 2x = 0$, $x = y$, $y^2 + 2y^2 - 4 \cdot y \cdot y = -7$, $y^2 + 2y^2 - 4y^2 = -7$,

$-y^2 = -7$, $y^2 = 7$, $y = \pm 2.6458$, $x = \pm 2.6458$

$(-2.6458, -2.6458)$ ← $(2.6458, 2.6458)$

(8) $f(x) = -\frac{13}{90}x^3 + \frac{199}{90}x^2 - \frac{142}{15}x + \frac{67}{5}$, $f'(x) = -\frac{13}{30}x^2 + \frac{199}{45}x - \frac{142}{15}$

$f(6) = 5$, $f'(6) = \frac{22}{15}$, $\left.\frac{df^{-1}}{dx}\right|_{(5,6)} = \frac{15}{22}$, $y = \frac{15}{22}x + b$,

$6 = \frac{15}{22}(5) + b$, $b = \frac{57}{22}$, $y = \frac{15}{22}x + \frac{57}{22}$ ←