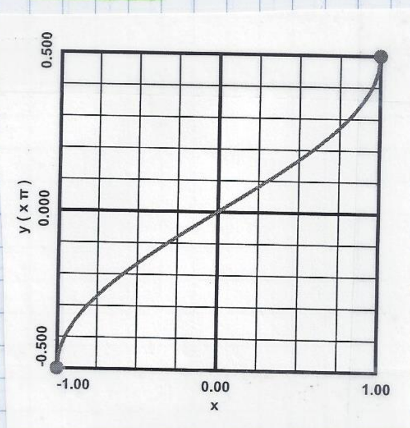
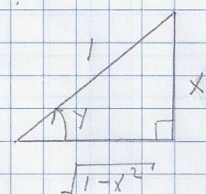
4.3. Derivatives of the Inverse Trig Functions

$y = \sin^{-1} x$





$y = \sin^{-1} x$

$x = \sin y$

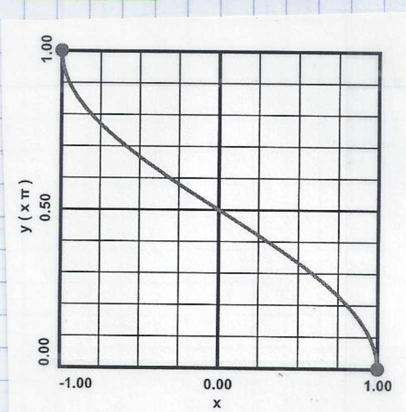
$\cos y = \sqrt{1-x^2}$

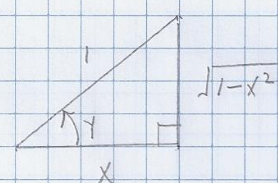
$x = \sin y$, $\dfrac{dx}{dx} = 1 = \dfrac{d \sin y}{dx} = \dfrac{d \sin y}{dy} \dfrac{dy}{dx}$,

$1 = \cos y \dfrac{dy}{dx}$, $\dfrac{dy}{dx} = \dfrac{1}{\cos y} \Rightarrow$

$$\boxed{\frac{d \sin^{-1} x}{dx} = \frac{1}{\sqrt{1-x^2}}}$$

$y = \cos^{-1} x$





$y = \cos^{-1} x$

$x = \cos y$

$\sin y = \sqrt{1-x^2}$

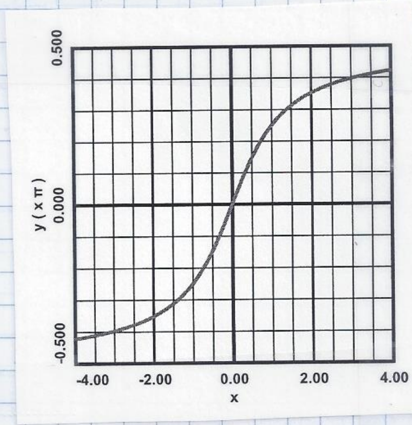
$x = \cos y$, $\dfrac{dx}{dx} = 1 = \dfrac{d \cos y}{dx} = \dfrac{d \cos y}{dy} \dfrac{dy}{dx}$

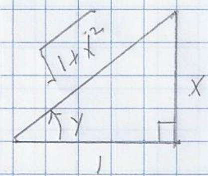
$1 = -\sin y \dfrac{dy}{dx}$, $\dfrac{dy}{dx} = -\dfrac{1}{\sin y} \Rightarrow$

$$\boxed{\frac{d \cos^{-1} x}{dx} = -\frac{1}{\sqrt{1-x^2}}}$$

4.3. Derivatives of the Inverse Trig Functions

$y = \tan^{-1} x$





$y = \tan^{-1} x$

$x = \tan y$

$\cos y = \dfrac{1}{\sqrt{1+x^2}}$

$\sec y = \sqrt{1+x^2}$ $\quad \sec^2 y = 1 + x^2$

$x = \tan y, \quad \dfrac{dx}{dx} = 1 = \dfrac{d\tan y}{dx} = \dfrac{d\tan y}{dy}\dfrac{dy}{dx}$

$1 = \sec^2 y \dfrac{dy}{dx}, \quad \dfrac{dy}{dx} = \dfrac{1}{\sec^2 y} = \dfrac{1}{1+x^2} \Rightarrow$

$$\boxed{\frac{d\tan^{-1} x}{dx} = \frac{1}{1+x^2}}$$

$\sec^{-1}$, $\csc^{-1}$ & $\cot^{-1}$

$x = \sec\theta \qquad \theta = \sec^{-1} x$

$\dfrac{1}{x} = \cos\theta \qquad \theta = \cos^{-1}\left(\dfrac{1}{x}\right) \Rightarrow \boxed{\sec^{-1} x = \cos^{-1}\left(\dfrac{1}{x}\right)}$

$x = \csc\theta \qquad \theta = \csc^{-1} x$

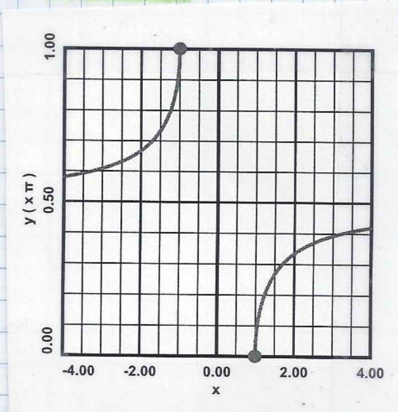
$\dfrac{1}{x} = \sin\theta \qquad \theta = \sin^{-1}\left(\dfrac{1}{x}\right) \Rightarrow \boxed{\csc^{-1} x = \sin^{-1}\left(\dfrac{1}{x}\right)}$

$x = \cot\theta \qquad \theta = \cot^{-1} x$

$\dfrac{1}{x} = \tan\theta \qquad \theta = \tan^{-1}\left(\dfrac{1}{x}\right) \Rightarrow \boxed{\cot^{-1} x = \tan^{-1}\left(\dfrac{1}{x}\right)}$

4.3. Derivatives of the Inverse Trig Functions

$y = \sec^{-1}x$



$\sec^{-1}x = \cos^{-1}\left(\frac{1}{x}\right)$ or

$\sec^{-1}x = \cos^{-1}g \; , \; g = \frac{1}{x} = x^{-1}$

$$\frac{d\sec^{-1}x}{dx} = \frac{d\cos^{-1}g}{dg}\,\frac{dg}{dx} = -\frac{1}{\sqrt{1-g^2}} \cdot -1x^{-2}$$

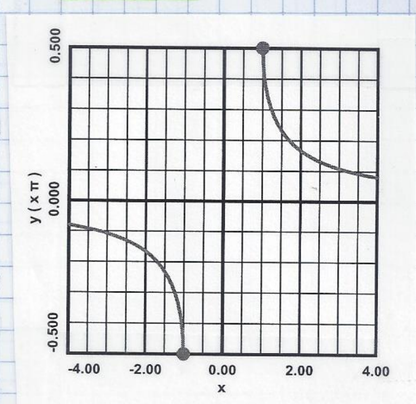
$$\frac{d\sec^{-1}x}{dx} = \frac{1}{x^2\sqrt{1-\left(\frac{1}{x}\right)^2}} = \frac{1}{x^2\sqrt{\frac{x^2}{x^2}-\frac{1}{x^2}}} =$$

$$= \frac{1}{\frac{|x|\cancel{|x|}}{\cancel{|x|}}\sqrt{x^2-1}} = \frac{1}{|x|\sqrt{x^2-1}}$$

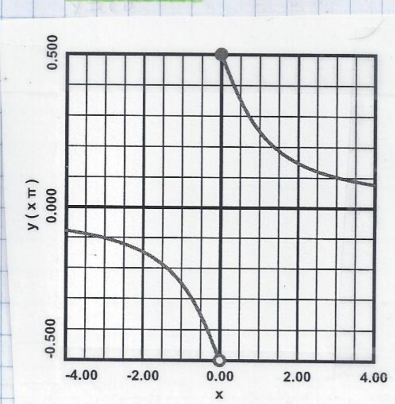
$$\boxed{\frac{d\sec^{-1}x}{dx} = \frac{1}{|x|\sqrt{x^2-1}}}$$

CLASS WORK

$y = \csc^{-1}x$



$y = \cot^{-1}x$



1) Given that $\csc^{-1}x = \sin^{-1}\left(\frac{1}{x}\right)$, use the Chain Rule to calculate $\frac{d\csc^{-1}x}{dx}$.

2) Given that $\cot^{-1}x = \tan^{-1}\left(\frac{1}{x}\right)$, use the Chain Rule to calculate $\frac{d\cot^{-1}x}{dx}$.

4.3. Derivatives of the Inverse Trig Functions

1) $\csc^{-1}x = \sin^{-1}g$, $g = \frac{1}{x} = x^{-1}$, $\frac{d\csc^{-1}x}{dx} = \frac{d\sin^{-1}g}{dg}\frac{dg}{dx} = \frac{1}{\sqrt{1-g^2}} \cdot -1x^{-2} =$

$$= -\frac{1}{x^2\sqrt{1-\left(\frac{1}{x}\right)^2}} = -\frac{1}{x^2\sqrt{\frac{x^2}{x^2}-\frac{1}{x^2}}} = -\frac{1}{\frac{|x|\cancel{|x|}}{\cancel{|x|}}\sqrt{x^2-1}} = -\frac{1}{|x|\sqrt{x^2-1}}$$

2) $\cot^{-1}x = \tan^{-1}g$, $g = \frac{1}{x} = x^{-1}$, $\frac{d\cot^{-1}x}{dx} = \frac{d\tan^{-1}g}{dg} \cdot \frac{dg}{dx} = \frac{1}{1+g^2} \cdot -1x^{-2} =$

$$= -\frac{1}{x^2\left[1+\left(\frac{1}{x}\right)^2\right]} = -\frac{1}{x^2\left(1+\frac{1}{x^2}\right)} = -\frac{1}{x^2+1} = -\frac{1}{1+x^2}$$