

4.4, Derivatives of Logarithmic Functions

1 OF 2

$f(x) = \ln x$

$$y = \ln x \quad e^y = x \quad \frac{de^y}{dx} = \frac{dx}{dx} = 1 \quad \frac{de^y}{dy} = \frac{de^y}{dy} \frac{dy}{dx} = e^y \frac{dy}{dx} = x \frac{d \ln x}{dx} = 1$$

$$\frac{d \ln x}{dx} = \frac{1}{x}$$

$f(x) = \log_b x$

$$y = \log_b x \quad b^y = x \quad \frac{db^y}{dx} = \frac{db^y}{dy} \frac{dy}{dx} = (\ln b) b^y \frac{dy}{dx} = (\ln b) x \frac{d \log_b x}{dx} = 1$$

$$\frac{d \log_b x}{dx} = \frac{1}{(\ln b) x}$$

Example. Calculate $f'(x)$ for

(a) $f(x) = x \ln x - x$ (b) $f(x) = \ln(2x^2 - 3x)$ (c) $f(x) = x^2 \log_3(2x+1)$

SOLUTION:

(a) $f'(x) = 1 \cdot \ln x + x \cdot \frac{1}{x} - 1 = \ln x + 1 - 1 = \ln x$ \leftarrow

(b) $f'(x) = \frac{1}{2x^2 - 3x} \cdot (4x - 3) = \frac{4x - 3}{2x^2 - 3x}$ \leftarrow

(c) $f'(x) = 2x \log_3(2x+1) + x^2 \cdot \frac{1}{(\ln 3)(2x+1)} \cdot 2$
 $= 2x \log_3(2x+1) + \frac{2x^2}{(\ln 3)(2x+1)}$ \leftarrow

4.4. Derivatives of Logarithmic Functions

2 of 2

CLASS WORK

Differentiate the functions.

(a) $f(x) = \frac{1}{3}x^3 \ln x - \frac{1}{9}x^3$ (b) $f(x) = \ln(5x^2 - 2x + 3)$

(c) $f(x) = (3x^2 + 7x) \log_5(7x^2 + 3x)$

SOLUTIONS

(a)
$$f'(x) = \frac{1}{3} \left[3x^2 \ln x + x^3 \cdot \frac{1}{x} \right] - \frac{1}{9} \cdot 3x^2 =$$
$$= x^2 \ln x + \frac{1}{3}x^2 - \frac{1}{3}x^2 = x^2 \ln x \quad \leftarrow$$

(b)
$$f'(x) = \frac{1}{5x^2 - 2x + 3} \cdot (10x - 2) = \frac{10x - 2}{5x^2 - 2x + 3} \quad \leftarrow$$

(c)
$$f'(x) = (6x + 7) \log_5(7x^2 + 3x) + (3x^2 + 7x) \cdot \frac{1}{(\ln 5)(7x^2 + 3x)} \cdot (14x + 3)$$
$$= (6x + 7) \log_5(7x^2 + 3x) + \frac{(3x^2 + 7x)(14x + 3)}{(\ln 5)(7x^2 + 3x)} =$$
$$= (6x + 7) \log_5(7x^2 + 3x) + \frac{42x^3 + 107x^2 + 21x}{(\ln 5)(7x^2 + 3x)} \quad \leftarrow$$