Derivatives Involving the Inverse Trigonometric Functions

1) $\dfrac{d}{dx}\left[x\sin^{-1}x + \sqrt{1-x^2}\right] = 1\cdot \sin^{-1}x + x\cdot \dfrac{1}{\sqrt{1-x^2}} + \dfrac{1}{\cancel{2}}(1-x^2)^{-1/2}\cdot -\cancel{2}x =$

$= \sin^{-1}x + \dfrac{x}{\sqrt{1-x^2}} - \dfrac{x}{\sqrt{1-x^2}} = \sin^{-1}x$ ◄

2) $\dfrac{d}{dx}\left[x\cos^{-1}x - \sqrt{1-x^2}\right] = 1\cdot \cos^{-1}x + x\cdot \dfrac{-1}{\sqrt{1-x^2}} - \dfrac{1}{\cancel{2}}(1-x^2)^{-1/2}\cdot -\cancel{2}x =$

$= \cos^{-1}x - \dfrac{x}{\sqrt{1-x^2}} + \dfrac{x}{\sqrt{1-x^2}} = \cos^{-1}x$ ◄

3) $\dfrac{d}{dx}\left[x\tan^{-1}x - \dfrac{1}{2}\ln(1+x^2)\right] = 1\cdot \tan^{-1}x + x\cdot \dfrac{1}{1+x^2} - \dfrac{1}{\cancel{2}}\cdot \dfrac{1}{1+x^2}\cdot \cancel{2}x =$

$= \tan^{-1}x + \dfrac{x}{1+x^2} - \dfrac{x}{1+x^2} = \tan^{-1}x$ ◄

4) $\dfrac{d}{dx}\left[x\cot^{-1}x + \dfrac{1}{2}\ln(1+x^2)\right] = 1\cdot \cot^{-1}x + x\cdot \dfrac{-1}{1+x^2} + \dfrac{1}{\cancel{2}}\cdot \dfrac{1}{(1+x^2)}\cdot \cancel{2}x =$

$= \cot^{-1}x - \dfrac{x}{1+x^2} + \dfrac{x}{1+x^2} = \cot^{-1}x$ ◄

5) For problems 5 and 6 we need $\dfrac{d}{dx}\ln\left(|x| + \sqrt{x^2-1}\right) =$

$= \dfrac{1}{|x| + \sqrt{x^2-1}}\cdot\left[\dfrac{x}{|x|} + \dfrac{1}{\cancel{2}}(x^2-1)^{-1/2}\cdot \cancel{2}x\right] =$

$= \dfrac{1}{|x| + \sqrt{x^2-1}}\left[\dfrac{x}{|x|} + \dfrac{x}{\sqrt{x^2-1}}\right] = \dfrac{x}{|x| + \sqrt{x^2-1}}\left[\dfrac{1}{|x|} + \dfrac{1}{\sqrt{x^2-1}}\right] =$

$= \dfrac{x}{|x| + \sqrt{x^2-1}}\left[\dfrac{1}{|x|}\cdot\dfrac{\sqrt{x^2-1}}{\sqrt{x^2-1}} + \dfrac{1}{\sqrt{x^2-1}}\cdot\dfrac{|x|}{|x|}\right] =$

$= \dfrac{x}{\cancel{|x| + \sqrt{x^2-1}}}\cdot\dfrac{\cancel{|x| + \sqrt{x^2-1}}}{|x|\sqrt{x^2-1}} = \dfrac{x}{|x|\sqrt{x^2-1}}$

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5) $\frac{d}{dx}\left[x\sec^{-1}x - \ln\left(|x| + \sqrt{x^2-1}\right)\right] = \frac{d}{dx}\left[x\sec^{-1}x\right] - \frac{d}{dx}\ln\left(|x| + \sqrt{x^2-1}\right) =$

$= 1\cdot\sec^{-1}x + x\cdot\frac{1}{|x|\sqrt{x^2-1}} - \frac{x}{|x|\sqrt{x^2-1}} = \sec^{-1}x$

6) $\frac{d}{dx}\left[x\csc^{-1}x + \ln\left(|x| + \sqrt{x^2-1}\right)\right] = \frac{d}{dx}\left[x\csc^{-1}x\right] + \frac{d}{dx}\ln\left(|x| + \sqrt{x^2-1}\right) =$

$= 1\cdot\csc^{-1}x + x\cdot\frac{-1}{|x|\sqrt{x^2-1}} + \frac{x}{|x|\sqrt{x^2-1}} = \csc^{-1}x$