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4.3. Derivatives of the Inverse Trig Functions

(1) $y=\cos^{-1}(x^2)$, $y' = -\frac{1}{\sqrt{1-(x^2)^2}}\cdot 2x = -\frac{2x}{\sqrt{1-x^4}}$

(5) $y=\sin^{-1} g$, $g=\frac{3}{t^2}=3t^{-2}$, $\frac{dg}{dt}=-6t^{-3}=-\frac{6}{t^3}$, $\frac{dy}{dt}=\frac{d\sin^{-1}g}{dg}\frac{dg}{dt}=\frac{1}{\sqrt{1-g^2}}\cdot -\frac{6}{t^3}=$

$$=-\frac{6}{t^3\sqrt{1-\frac{9}{t^4}}} = \frac{-6}{t^3\sqrt{\frac{t^4}{t^4}-\frac{9}{t^4}}} = \frac{-6}{\frac{t^3}{t^2}\sqrt{t^4-9}} = \frac{-6}{t\sqrt{t^4-9}}$$

(6) $y = s\sqrt{1-s^2}+\cos^{-1}s$, $y' = 1\cdot\sqrt{1-s^2}+s\cdot\frac{1}{\cancel{2}}(1-s^2)^{-1/2}\cdot -\cancel{2}s - \frac{1}{\sqrt{1-s^2}} =$

$$=\frac{\sqrt{1-s^2}}{1}\cdot\frac{\sqrt{1-s^2}}{\sqrt{1-s^2}} - \frac{s^2}{\sqrt{1-s^2}} - \frac{1}{\sqrt{1-s^2}} = \frac{\cancel{1}-s^2-s^2-\cancel{1}}{\sqrt{1-s^2}} = -\frac{2s^2}{\sqrt{1-s^2}}$$

(7) $y = x\sin^{-1}x+\sqrt{1-x^2}$, $y' = 1\cdot\sin^{-1}x + x\cdot\frac{1}{\sqrt{1-x^2}} + \frac{1}{\cancel{2}}(1-x^2)^{-1/2}\cdot -\cancel{2}x =$

$$= \sin^{-1}x + \frac{x}{\sqrt{1-x^2}} - \frac{x}{\sqrt{1-x^2}} = \sin^{-1}x$$

(15) $y=\csc^{-1}g$, $g=x^2+1$, $\frac{dy}{dx}=\frac{d\csc^{-1}g}{dg}\cdot\frac{dg}{dx} = -\frac{1}{|g|\sqrt{g^2-1}}\cdot(2x)=$

$$=-\frac{2x}{|x^2+1|\sqrt{(x^2+1)^2-1}} = -\frac{2x}{|x^2+1|\sqrt{x^4+2x^2}} = \frac{-2x}{|x^2+1|\cdot|x|\sqrt{x^2+2}} =$$

$$=\frac{-2}{|x^3+x|\sqrt{x^2+2}}$$

(17) $y=\sec^{-1}g$, $g=\frac{1}{t}$, $\frac{dy}{dt}=\frac{1}{|g|\sqrt{g^2-1}}\cdot -1t^{-2} = -\frac{1}{t^2\left|\frac{1}{t}\right|\sqrt{\frac{1}{t^2}-\frac{t^2}{t^2}}} =$

$$=-\frac{1}{t^2\left|\frac{1}{t}\right|\cdot\left|\frac{1}{t}\right|\sqrt{1-t^2}} = \frac{-1}{\sqrt{1-t^2}}$$

(18) $y=\cot^{-1}g$, $g=\sqrt{t}$, $y' = -\frac{1}{1+g^2}\cdot\frac{1}{2}t^{-1/2} = -\frac{1}{2\sqrt{t}\,(1+|t|)}$

(21)

$y = \tan^{-1}\sqrt{x^2-1} + \csc^{-1}x$, $g = \sqrt{x^2-1}$, $\frac{dg}{dx} = \frac{1}{\cancel{2}}(x^2-1)^{-1/2}\cdot\cancel{2}x =$

$= \frac{x}{\sqrt{x^2-1}}$. $\frac{dy}{dx} = \frac{d\tan^{-1}g}{dg}\frac{dg}{dx} - \frac{1}{|x|\sqrt{x^2-1}} = \frac{1}{1+g^2}\cdot\frac{x}{\sqrt{x^2-1}} - \frac{1}{|x|\sqrt{x^2-1}} =$

$= \frac{1}{x^2}\cdot\frac{x}{\sqrt{x^2-1}} - \frac{x}{|x|\sqrt{x^2-1}} = \frac{1}{x\sqrt{x^2-1}} - \frac{|x|}{x\sqrt{x^2-1}} = \frac{1-|x|}{x\sqrt{x^2-1}}$ ◂

pg. 196 4.4. Derivatives of Exponential Functions

(7) $y = xe^2 - e^x$, $y' = e^2 - e^x$ ◂

(8) $y = x^2e^x - xe^x$, $y' = 2xe^x + x^2e^x - (1\cdot e^x + x\cdot e^x) = (2x+x^2-1-x)e^x =$

$= (x^2+x-1)e^x$ ◂

(9) $y = e^{\sqrt{x}}$, $y' = e^{\sqrt{x}}\cdot\frac{1}{2}x^{-1/2} = \frac{e^{\sqrt{x}}}{2\sqrt{x}}$ ◂

(11) $y = 8^x$, $y' = (\ln 8)8^x$ ◂

(12) $y = 9^{-x} = \frac{1}{9^x}$ $y' = -\frac{1}{(9^x)^{\cancel{2}}}\cdot(\ln 9)\cancel{9^x} = -\frac{\ln 9}{9^x}$ ◂

(13) $y = 3^{\csc x}$, $y' = (\ln 3)3^{\csc x}\cdot -\csc x\cot x = -(\ln 3)\csc x\cot x\cdot 3^{\csc x}$ ◂

(33) $y = x^{\pi}$, $y' = \pi x^{\pi-1} = \frac{\pi x^{\pi}}{x}$ ◂

(34) $y = x^{1+\sqrt{2}}$, $y' = (1+\sqrt{2})x^{\sqrt{2}}$ ◂

(55) (c) $\frac{d2^x}{dx} = (\ln 2)2^x = \lim_{h\to 0}\frac{2^{x+h}-2^x}{h} = \lim_{h\to 0}\frac{2^x2^h-2^x}{h} = 2^x\cdot\lim_{h\to 0}\frac{2^h-1}{h} \Rightarrow$

$\lim_{h\to 0}\frac{2^h-1}{h} = \ln 2$ ◂

pg. 186 4.4. Derivatives of Logarithmic Functions

(15) $y = \ln x^2 = 2\ln x$, $y' = \frac{2}{x}$ ◂

(16) $y = (\ln x)^2$, $y' = 2\ln x\cdot\frac{1}{x} = \frac{\ln x^2}{x}$ ◂

(17) $y = \ln\left(\frac{1}{x}\right) = \ln x^{-1} = -\ln x$, $y' = -\frac{1}{x}$

(21) $y = \log_4 x^2 = 2\log_4 x$, $y' = 2 \cdot \frac{1}{(\ln 4)x} = \frac{2}{(\ln 2^2)x} = \frac{2}{2\ln 2 \cdot x} = \frac{1}{(\ln 2)x}$

(24) $y = \frac{1}{\log_2 x}$, $y' = -1(\log_2 x)^{-2} \cdot \frac{1}{(\ln 2)x} = \frac{-1}{(\ln 2)(\log_2 x)^2 x}$

(26) $y = \log_3(1 + x\ln 3)$, $y' = \frac{\ln 3}{(\ln 3)(1 + x\ln 3)} = \frac{1}{1 + x\ln 3}$

Supplemental:

(1)

(a) $\sinh x = \frac{1}{2}(e^x - e^{-x})$, $\frac{d\sinh x}{dx} = \frac{1}{2}(e^x - e^{-x} \cdot -1) = \frac{1}{2}(e^x + e^{-x}) = \cosh x$

(b) $\cosh x = \frac{1}{2}(e^x + e^{-x})$, $\frac{d\cosh x}{dx} = \frac{1}{2}(e^x + e^{-x} \cdot -1) = \frac{1}{2}(e^x - e^{-x}) = \sinh x$

Supplemental: 4.4. Derivatives Involving Absolute Values

(2)

(a) $y = \ln|\sec x|$, $y' = \frac{1}{|\sec x|} \cdot \frac{|\sec x|}{\sec x}$, $\sec x \tan x = \tan x$

(b) $y = \ln|\csc x - \cot x|$, $y' = \frac{1}{|\csc x - \cot x|} \cdot \frac{|\csc x - \cot x|}{\csc x - \cot x} \cdot (-\csc x \cot x + \csc^2 x) =$

$= \frac{\csc x(\csc x - \cot x)}{\csc x - \cot x} = \csc x$

(3) $f(x) = \frac{3x^2 - 5x - 2}{|x-2|} = \frac{u}{v}$ $f'(x) = \frac{u'v - uv'}{v^2} = \frac{(6x-5)|x-2| - (3x^2 - 5x - 2)\frac{|x-2|}{(x-2)}}{(x-2)^2} =$

$= \frac{|x-2|\left[\frac{(6x-5)(x-2)}{(x-2)} - \frac{3x^2-5x-2}{(x-2)}\right]}{(x-2)^2} = \frac{|x-2|\left[(6x-5)(x-2) - 3x^2 + 5x + 2\right]}{(x-2)^3} =$

$= \frac{|x-2|(6x^2 - 12x - \cancel{5x} + 10 - 3x^2 + \cancel{5x} + 2)}{(x-2)^3} = \frac{|x-2|(3x^2 - 12x + 12)}{(x-2)^3} =$

$= \frac{|x-2| \cdot 3(x^2 - 4x + 4)}{(x-2)^3} = \frac{|x-2| \cdot 3(x-2)^2}{(x-2)^3} = 3 \cdot \frac{|x-2|}{x-2}$

pg. 186 4.4. Logarithmic Differentiation

(43) $y = (\sin x)^x$, $\ln y = \ln(\sin x)^x = x\ln(\sin x)$, $\frac{d\ln y}{dx} = \frac{d\ln y}{dy}\frac{dy}{dx} = \frac{1}{y}\frac{dy}{dx} =$

$= 1 \cdot \ln(\sin x) + x \cdot \frac{1}{\sin x} \cdot \cos x = x\cot x + \ln(\sin x)$,

$$\frac{dy}{dx} = y\left[x\cot x + \ln(\sin x)\right] = \left[x\cot x + \ln(\sin x)\right](\sin x)^{x}$$

(44) $y = x^{\tan x}$, $\ln y = \ln x^{\tan x} = \tan x \cdot \ln x$, $\frac{1}{y}\frac{dy}{dx} = \sec^2 x \cdot \ln x + \frac{\tan x}{x} =$

$= \frac{\tan x + x\sec^2 x \cdot \ln x}{x}$, $\frac{dy}{dx} = x^{\tan x} \cdot \frac{\tan x + x\sec^2 x \ln x}{x} =$

$= (\tan x + x\sec^2 x \ln x)\, x^{\tan x - 1}$

(47) $y = x^{\ln x}$, $\ln y = \ln x^{\ln x} = (\ln x)^2$, $\frac{1}{y}\frac{dy}{dx} = 2\ln x \cdot \frac{1}{x}$, $\frac{dy}{dx} = \frac{2\ln x}{x} \cdot x^{\ln x} =$

$= \ln x^2 \cdot x^{\ln x - 1}$

Supplemental:

(4) (a) $y = x^{2x}$, $\ln y = \ln x^{2x} = 2x \cdot \ln x$, $\frac{1}{y}\frac{dy}{dx} = 2\left(1 \cdot \ln x + x \cdot \frac{1}{x}\right) = 2(1 + \ln x)$

$\frac{dy}{dx} = 2(1+\ln x)\, y = 2(1 + \ln x) \cdot x^{2x}$

(b) $y = (4x)^{(3x)}$, $\ln y = \ln(4x)^{(3x)} = 3x\ln(4x)$,

$\frac{1}{y}\frac{dy}{dx} = 3\left[1 \cdot \ln(4x) + x \cdot \frac{1}{\cancel{4}x} \cdot 4\right] = 3\left[1 + \ln(4x)\right]$

$\frac{dy}{dx} = 3\left[1 + \ln(4x)\right] y = 3\left[1 + \ln(4y)\right](4x)^{(3x)}$