Quiz #6 Study Guide

(1) $f(x) = \sec^{-1} g$, $g = \frac{1}{\sqrt{x}} = x^{-1/2}$, $\frac{dg}{dx} = -\frac{1}{2}x^{-3/2} = -\frac{1}{2\sqrt{x^3}}$

$$\frac{df}{dx} = \frac{d\sec^{-1} g}{dg} \cdot \frac{dg}{dx} = \frac{1}{|g|\sqrt{g^2-1}} \cdot -\frac{1}{2\sqrt{x^3}} = \frac{1}{\left|\frac{1}{\sqrt{x}}\right|\sqrt{\left(\frac{1}{\sqrt{x}}\right)^2 - 1}} \cdot -\frac{1}{2\sqrt{x^3}} =$$

$$= \frac{1}{\frac{1}{\sqrt{x}}\sqrt{\frac{1}{x} - \frac{x}{x}}} \cdot -\frac{1}{2\sqrt{x^3}} = \frac{1}{\frac{1}{x}\sqrt{1-x}} \cdot -\frac{1}{2\sqrt{x^3}} = -\frac{x}{2\sqrt{x^3}\sqrt{1-x}}$$

(2) $f(x) = \tan^{-1} g$, $g = \frac{7}{x^2} = 7x^{-2}$, $\frac{dg}{dx} = -14x^{-3} = -\frac{14}{x^3}$,

$$\frac{df}{dx} = \frac{d\tan^{-1} g}{dg} \cdot \frac{dg}{dx} = \frac{1}{1+g^2} \cdot -\frac{14}{x^3} = \frac{1}{1+\left(\frac{7}{x^2}\right)^2} \cdot -\frac{14}{x^3} = \frac{1}{\frac{x^4}{x^4} + \frac{49}{x^4}} \cdot -\frac{14}{x^3} =$$

$$= -\frac{14x}{x^4+49}$$

(3) $f(x) = (x^4 - 4x^3 + 12x^2 - 24x + 24)e^x$

$f'(x) = (4x^3 - 12x^2 + 24x - 24)e^x + (x^4 - 4x^3 + 12x^2 - 24x + 24)e^x =$

$= x^4 e^x$

(4) $f(x) = \frac{4x+7}{5^x} = \frac{u}{v}$, $f'(x) = \frac{u'v - uv'}{v^2} = \frac{4 \cdot 5^x - (4x+7)(\ln 5)5^x}{(5^x)^2} =$

$$= \frac{4 - (\ln 5)(4x+7)}{5^x}$$

(5) $f(x) = \frac{\log_4 x}{x^5} = \frac{u}{v}$ $f'(x) = \frac{u'v - uv'}{v^2} = \frac{\frac{1}{(\ln 4)x} \cdot x^5 - \log_4 x \cdot \frac{5x^4(\ln 4)}{(\ln 4)}}{x^{10}} =$

$$= \frac{x^4 - 5(\ln 4)x^4 \log_4 x}{(\ln 4)x^{10}} = \frac{1 - 5(\ln 4)\log_4 x}{(\ln 4)x^6}$$

(6) $f'(x) = \cot^{-1} x + x \cdot \frac{-1}{1+x^2} + \frac{1}{\sqrt{1+x^2}} \cdot \frac{1}{2}(1+x^2)^{-1/2} \cdot 2x =$

$= \cot^{-1} x - \frac{x}{1+x^2} + \frac{x}{1+x^2} = \cot^{-1} x$

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(7) $f(x) = \dfrac{\ln|x^3-8|}{x} = \dfrac{u}{v}$ $\quad u' = \dfrac{1}{|x^3-8|}\cdot\dfrac{|x^3-8|}{x^3-8}\cdot 3x^2 = \dfrac{3x^2}{x^3-8}$

$$f'(x) = \frac{u'v - uv'}{v^2} = \frac{\frac{3x^2}{x^3-8}\cdot x - \ln|x^3-8|\cdot 1}{x^2} \cdot \frac{(x^3-8)}{(x^3-8)} =$$

$$= \frac{3x^3 - (x^3-8)\ln|x^3-8|}{x^2(x^3-8)}$$

(8) $f(x) = \dfrac{3x^2-5x-28}{|x-4|} = \dfrac{u}{v}$ $\quad f'(x) = \dfrac{u'v-uv'}{v^2} = \dfrac{(6x-5)|x-4| - (3x^2-5x-28)\cdot\frac{|x-4|}{x-4}}{(x-4)^2} =$

$$= \frac{\frac{(6x-5)(x-4)|x-4|}{(x-4)} - \frac{(3x^2-5x-28)|x-4|}{(x-4)}}{(x-4)^2} =$$

$$= \frac{|x-4|}{(x-4)^3}\left(6x^2-24x-5x+20-3x^2+5x+28\right) = \frac{|x-4|}{(x-4)^3}\cdot(3x^2-24x+48) =$$

$$= \frac{|x-4|}{(x-4)^3}\cdot 3(x^2-8x+16) = \frac{|x-4|}{(x-4)^3}\cdot 3(x-4)^2 = 3\cdot\frac{|x-4|}{x-4}$$

(9) $f(x) = x^{7x}$, $\ln f = \ln x^{7x} = 7x\ln x$, $\dfrac{d\ln f}{dx} = \dfrac{d\ln f}{df}\dfrac{df}{dx} = \dfrac{1}{f}\dfrac{df}{dx} =$

$$= 7\left(1\cdot\ln x + x\cdot\frac{1}{x}\right) = 7(1+\ln x), \quad \frac{df}{dx} = 7(1+\ln x)f = 7(1+\ln x)x^{7x}$$

(10) $f(x) = (9x)^{2x}$, $\ln f = \ln(9x)^{2x} = 2x\ln(9x)$

$$\frac{1}{f}\frac{df}{dx} = 2\left[1\cdot\ln(9x) + x\cdot\frac{1}{9x}\cdot 9\right] = 2\left[1+\ln(9x)\right]$$

$$\frac{df}{dx} = 2\left[1+\ln(9x)\right]f = 2\left[1+\ln(9x)\right](9x)^{2x}$$