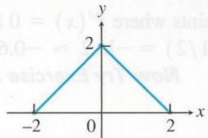
Section 5.1 Exercises

In Exercises 1–4, find the extreme values and where they occur.

1.



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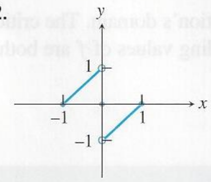

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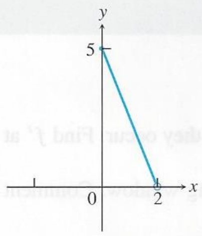

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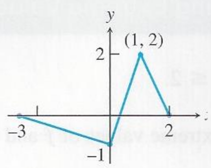
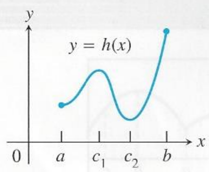


In Exercises 5–10, identify each x value at which any absolute extreme value occurs. Explain how your answer is consistent with the Extreme Value Theorem.

5.



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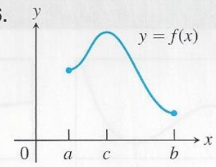

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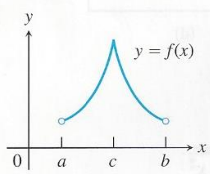

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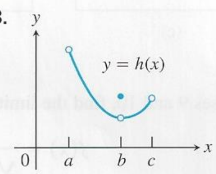

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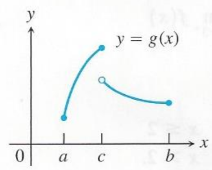
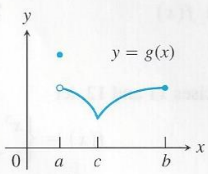


10.



In Exercises 11–18, use analytic methods to find the extreme values of the function on the interval and where they occur. Identify any critical points that are *not* stationary points.

11. $f(x) = \dfrac{1}{x} + \ln x, \quad 0.5 \le x \le 4$

12. $g(x) = e^{-x}, \quad -1 \le x \le 1$

13. $h(x) = \ln (x + 1), \quad 0 \le x \le 3$

14. $k(x) = e^{-x^2}, \quad -\infty < x < \infty$

15. $f(x) = \sin\left(x + \dfrac{\pi}{4}\right), \quad 0 \le x \le \dfrac{7\pi}{4}$

16. $g(x) = \sec x, \quad -\dfrac{\pi}{2} < x < \dfrac{3\pi}{2}$

17. $f(x) = x^{2/5}, \quad -3 \le x < 1$

18. $f(x) = x^{3/5}, \quad -2 < x \le 3$

In Exercises 19–30, find the extreme values of the function and where they occur.

19. $y = 2x^2 - 8x + 9$

20. $y = x^3 - 2x + 4$

21. $y = x^3 + x^2 - 8x + 5$

22. $y = x^3 - 3x^2 + 3x - 2$

23. $y = \sqrt{x^2 - 1}$

24. $y = \dfrac{1}{x^2 - 1}$

25. $y = \dfrac{1}{\sqrt{1 - x^2}}$

26. $y = \dfrac{1}{\sqrt[3]{1 - x^2}}$

27. $y = \sqrt{3 + 2x - x^2}$

28. $y = \dfrac{3}{2}x^4 + 4x^3 - 9x^2 + 10$

29. $y = \dfrac{x}{x^2 + 1}$

30. $y = \dfrac{x + 1}{x^2 + 2x + 2}$

Group Activity In Exercises 31–34, find the extreme values of the function on the interval and where they occur.

31. $f(x) = |x - 2| + |x + 3|, \quad -5 \le x \le 5$

32. $g(x) = |x - 1| - |x - 5|, \quad -2 \le x \le 7$

33. $h(x) = |x + 2| - |x - 3|, \quad -\infty < x < \infty$

34. $k(x) = |x + 1| + |x - 3|, \quad -\infty < x < \infty$

In Exercises 35–42, identify the critical points and determine the local extreme values. Identify which critical points are *not* stationary points.

35. $y = x^{2/3}(x + 2)$

36. $y = x^{2/3}(x^2 - 4)$

37. $y = x\sqrt{4 - x^2}$

38. $y = x^2\sqrt{3 - x}$

39. $y = \begin{cases} 4 - 2x, & x \le 1 \\ x + 1, & x > 1 \end{cases}$

40. $y = \begin{cases} 3 - x, & x < 0 \\ 3 + 2x - x^2, & x \ge 0 \end{cases}$

41. $y = \begin{cases} -x^2 - 2x + 4, & x \le 1 \\ -x^2 + 6x - 4, & x > 1 \end{cases}$

42. $y = \begin{cases} -\dfrac{1}{4}x^2 - \dfrac{1}{2}x + \dfrac{15}{4}, & x \le 1 \\ x^3 - 6x^2 + 8x, & x > 1 \end{cases}$

43. Writing to Learn The function

$$V(x) = x(10 - 2x)(16 - 2x), \quad 0 < x < 5$$

models the volume of a box.

(a) Find the extreme values of V.

(b) Interpret any values found in (a) in terms of volume of the box.

(c) Support your analytic answer to part (a) graphically.

44. Writing to Learn The function

$$P(x) = 2x + \frac{200}{x}, \quad 0 < x < \infty,$$

models the perimeter of a rectangle of dimensions x by $100/x$.

(a) Find any extreme values of P.

(b) Give an interpretation in terms of perimeter of the rectangle for any values found in (a).

Standardized Test Questions

45. True or False If $f(c)$ is a local maximum of a continuous function f on an open interval (a, b), then $f'(c) = 0$. Justify your answer.

46. True or False If m is a local minimum and M is a local maximum of a continuous function f on (a, b), then $m < M$. Justify your answer.

47. Multiple Choice Which of the following values is the absolute maximum of the function $f(x) = 4x - x^2 + 6$ on the interval $[0, 4]$?

(A) 0 (B) 2 (C) 4 (D) 6 (E) 10

48. Multiple Choice If f is a continuous, decreasing function on $[0, 10]$ with a critical point at $(4, 2)$, which of the following statements *must be false*?

(A) $f(10)$ is an absolute minimum of f on $[0, 10]$.

(B) $f(4)$ is neither a relative maximum nor a relative minimum.

(C) $f'(4)$ does not exist.

(D) $f'(4) = 0$

(E) $f'(4) < 0$

49. Multiple Choice Which of the following functions has exactly two local extrema on its domain?

(A) $f(x) = |x - 2|$

(B) $f(x) = x^3 - 6x + 5$

(C) $f(x) = x^3 + 6x - 5$

(D) $f(x) = \tan x$

(E) $f(x) = x + \ln x$

50. Multiple Choice If an even function f with domain all real numbers has a local maximum at $x = a$, then $f(-a)$

(A) is a local minimum.

(B) is a local maximum.

(C) is both a local minimum and a local maximum.

(D) could be either a local minimum or a local maximum.

(E) is neither a local minimum nor a local maximum.

Explorations

In Exercises 51 and 52, give reasons for your answers.

51. Writing to Learn Let $f(x) = (x - 2)^{2/3}$.

(a) Does $f'(2)$ exist?

(b) Show that the only local extreme value of f occurs at $x = 2$.

(c) Does the result in (b) contradict the Extreme Value Theorem?

(d) Repeat parts (a) and (b) for $f(x) = (x - a)^{2/3}$, replacing 2 by a.

52. Writing to Learn Let $f(x) = |x^3 - 9x|$.

(a) Does $f'(0)$ exist?

(b) Does $f'(3)$ exist?

(c) Does $f'(-3)$ exist?

(d) Determine all extrema of f.

Extending the Ideas

53. *Cubic Functions* Consider the cubic function

$$f(x) = ax^3 + bx^2 + cx + d.$$

(a) Show that f can have 0, 1, or 2 critical points. Give examples and graphs to support your argument.

(b) How many local extreme values can f have?

54. *Proving Theorem 2* Assume that the function f has a local maximum value at the interior point c of its domain and that $f'(c)$ exists.

(a) Show that there is an open interval containing c such that $f(x) - f(c) \le 0$ for all x in the open interval.

(b) **Writing to Learn** Now explain why we may say

$$\lim_{x \to c^+} \frac{f(x) - f(c)}{x - c} \le 0.$$

(c) **Writing to Learn** Now explain why we may say

$$\lim_{x \to c^-} \frac{f(x) - f(c)}{x - c} \ge 0.$$

(d) **Writing to Learn** Explain how parts (b) and (c) allow us to conclude $f'(c) = 0$.

(e) **Writing to Learn** Give a similar argument if f has a local minimum value at an interior point.

55. *Functions with No Extreme Values at Endpoints*

(a) Graph the function

$$f(x) = \begin{cases} \sin\dfrac{1}{x}, & x > 0 \\ 0, & x = 0. \end{cases}$$

Explain why $f(0) = 0$ is not a local extreme value of f.

(b) **Group Activity** Construct a function of your own that fails to have an extreme value at a domain endpoint.