## Section 5.3 Exercises

In Exercises 1–6, use the **First Derivative Test** to determine the local extreme values of the function, and identify any absolute extrema. Support your answers graphically.

**1.** $y = x^2 - x - 1$

**2.** $y = -2x^3 + 6x^2 - 3$

**3.** $y = 2x^4 - 4x^2 + 1$

**4.** $y = xe^{1/x}$

**5.** $y = x\sqrt{8 - x^2}$

**6.** $y = \begin{cases} 3 - x^2, & x < 0 \\ x^2 + 1, & x \geq 0 \end{cases}$

In Exercises 7–12, use the Concavity Test to determine the intervals on which the graph of the function is **(a)** concave up and **(b)** concave down.

**7.** $y = 4x^3 + 21x^2 + 36x - 20$

**8.** $y = -x^4 + 4x^3 - 4x + 1$

**9.** $y = 2x^{1/5} + 3$

**10.** $y = 5 - x^{1/3}$

**11.** $y = \begin{cases} 2x, & x < 1 \\ 2 - x^2, & x \geq 1 \end{cases}$

**12.** $y = e^x, \quad 0 \leq x \leq 2\pi$

In Exercises 13–20, find all points of inflection of the function.

**13.** $y = xe^x$

**14.** $y = x\sqrt{9 - x^2}$

**15.** $y = \tan^{-1} x$

**16.** $y = x^3(4 - x)$

**17.** $y = x^{1/3}(x - 4)$

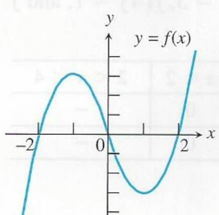
**18.** $y = x^{1/2}(x + 3)$

**19.** $y = \dfrac{x^3 - 2x^2 + x - 1}{x - 2}$

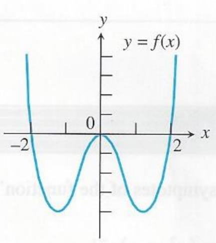
**20.** $y = \dfrac{x}{x^2 + 1}$

In Exercises 21 and 22, use the graph of the function $f$ to estimate where **(a)** $f'$ and **(b)** $f''$ are 0, positive, and negative.

**21.**

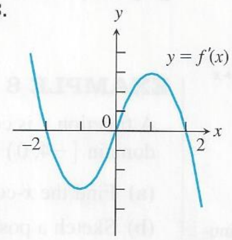



**22.**

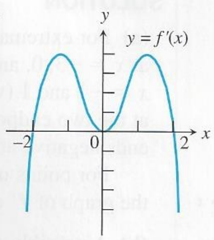



In Exercises 23 and 24, use the graph of the function $f'$ to estimate the intervals on which the function $f$ is **(a)** increasing or **(b)** decreasing. Also, **(c)** estimate the $x$-coordinates of all local extreme values.

**23.**




**24.**




In Exercises 25–28, a particle is moving along the $x$-axis with position function $x(t)$. Find the **(a)** velocity and **(b)** acceleration, and **(c)** describe the motion of the particle for $t \geq 0$.

**25.** $x(t) = t^2 - 4t + 3$

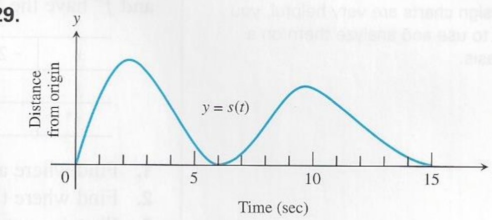
**26.** $x(t) = 6 - 2t - t^2$

**27.** $x(t) = t^3 - 3t + 3$

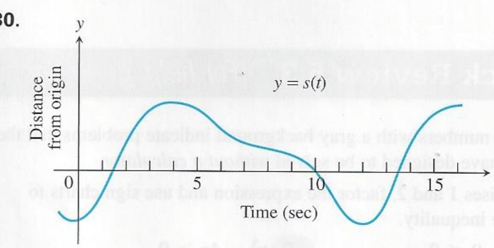
**28.** $x(t) = 3t^2 - 2t^3$

In Exercises 29 and 30, the graph of the position function $y = s(t)$ of a particle moving along a line is given. At approximately what times is the particle's **(a)** velocity equal to zero? **(b)** acceleration equal to zero?

**29.**




**30.**



In Exercises 31–36, use the Second Derivative Test to find the local extrema for the function.

**31.** $y = 3x - x^3 + 5$

**32.** $y = x^5 - 80x + 100$

**33.** $y = x^3 + 3x^2 - 2$

**34.** $y = 3x^5 - 25x^3 + 60x + 20$

**35.** $y = xe^x$

**36.** $y = xe^{-x}$

In Exercises 37 and 38, use the derivative of the function $y = f(x)$ to find the points at which $f$ has a

**(a)** local maximum, **(b)** local minimum, or

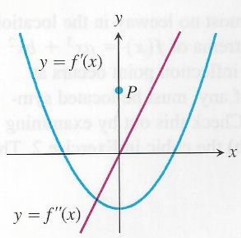
**(c)** point of inflection.

**37.** $y' = (x - 1)^2(x - 2)$

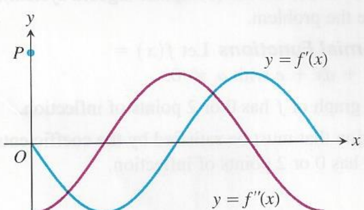
**38.** $y' = (x - 1)^2(x - 2)(x - 4)$

Exercises 39 and 40 show the graphs of the first and second derivatives of a function $y = f(x)$. Copy the figure and add a sketch of a possible graph of $f$ that passes through the point $P$.

**39.**




**40.**




**41. Writing to Learn** If $f(x)$ is a differentiable function and $f'(c) = 0$ at an interior point $c$ of $f$'s domain, must $f$ have a local maximum or minimum at $x = c$? Explain.

**42. Writing to Learn** If $f(x)$ is a twice-differentiable function and $f''(c) = 0$ at an interior point $c$ of $f$'s domain, must $f$ have an inflection point at $x = c$? Explain.

**43. *Connecting f and f'*** Sketch a smooth curve $y = f(x)$ through the origin with the properties that $f'(x) < 0$ for $x < 0$ and $f'(x) > 0$ for $x > 0$.

**44. *Connecting f and f''*** Sketch a smooth curve $y = f(x)$ through the origin with the properties that $f''(x) < 0$ for $x < 0$ and $f''(x) > 0$ for $x > 0$.

**45. *Connecting f, f', and f''*** Sketch a continuous curve $y = f(x)$ with the following properties. Label coordinates where possible.

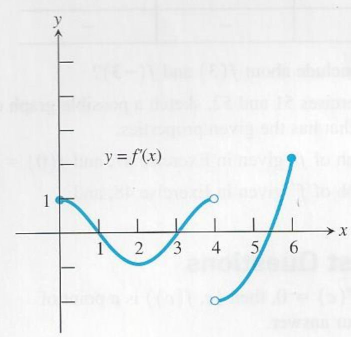
| | |
|---|---|
| $f(-2) = 8$ | $f'(x) > 0$ for $\lvert x\rvert > 2$ |
| $f(0) = 4$ | $f'(x) < 0$ for $\lvert x\rvert < 2$ |
| $f(2) = 0$ | $f''(x) < 0$ for $x < 0$ |
| $f'(2) = f'(-2) = 0$ | $f''(x) > 0$ for $x > 0$ |

**46. *Using Behavior to Sketch*** Sketch a continuous curve $y = f(x)$ with the following properties. Label coordinates where possible.

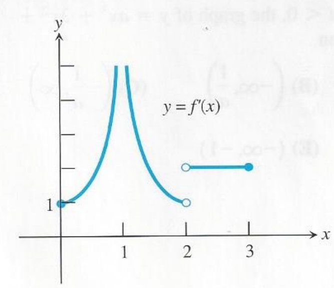
| $x$ | $y$ | Curve |
|---|---|---|
| $x < 2$ | | falling, concave up |
| 2 | 1 | horizontal tangent |
| $2 < x < 4$ | | rising, concave up |
| 4 | 4 | inflection point |
| $4 < x < 6$ | | rising, concave down |
| 6 | 7 | horizontal tangent |
| $x > 6$ | | falling, concave down |

In Exercises 47 and 48, use the graph of $f'$ to estimate the intervals on which the function $f$ is **(a)** increasing or **(b)** decreasing. Also, **(c)** estimate the $x$-coordinates of all local extreme values. (Assume that the function $f$ is continuous, even at the points where $f'$ is undefined.)

**47.** The domain of $f'$ is $[0, 4) \cup (4, 6]$.




**48.** The domain of $f'$ is $[0, 1) \cup (1, 2) \cup (2, 3]$.



**Group Activity** In Exercises 49 and 50, do the following.

(a) Find the absolute extrema of $f$ and where they occur.

(b) Find any points of inflection.

(c) Sketch a possible graph of $f$.

**49.** $f$ is continuous on $[0, 3]$ and satisfies the following.

| $x$ | 0 | 1 | 2 | 3 |
|---|---|---|---|---|
| $f$ | 0 | 2 | 0 | $-2$ |
| $f'$ | 3 | 0 | does not exist | $-3$ |
| $f''$ | 0 | $-1$ | does not exist | 0 |

| $x$ | $0 < x < 1$ | $1 < x < 2$ | $2 < x < 3$ |
|---|---|---|---|
| $f$ | + | + | $-$ |
| $f'$ | + | $-$ | $-$ |
| $f''$ | $-$ | $-$ | $-$ |

**50.** $f$ is an even function, continuous on $[-3, 3]$, and satisfies the following.

| $x$ | 0 | 1 | 2 |
|---|---|---|---|
| $f$ | 2 | 0 | $-1$ |
| $f'$ | does not exist | 0 | does not exist |
| $f''$ | does not exist | 0 | does not exist |

| $x$ | $0 < x < 1$ | $1 < x < 2$ | $2 < x < 3$ |
|---|---|---|---|
| $f$ | + | $-$ | $-$ |
| $f'$ | $-$ | $-$ | + |
| $f''$ | + | $-$ | $-$ |

(d) What can you conclude about $f(3)$ and $f(-3)$?

**Group Activity** In Exercises 51 and 52, sketch a possible graph of a continuous function $f$ that has the given properties.

**51.** Domain $[0, 6]$, graph of $f'$ given in Exercise 47, and $f(0) = 2$.

**52.** Domain $[0, 3]$, graph of $f'$ given in Exercise 48, and $f(0) = -3$.

## Standardized Test Questions

**53. True or False** If $f''(c) = 0$, then $(c, f(c))$ is a point of inflection. Justify your answer.

**54. True or False** If $f'(c) = 0$ and $f''(c) < 0$, then $f(c)$ is a local maximum. Justify your answer.

**55. Multiple Choice** If $a < 0$, the graph of $y = ax^3 + 3x^2 + 4x + 5$ is concave up on

(A) $\left(-\infty, -\frac{1}{a}\right)$ (B) $\left(-\infty, \frac{1}{a}\right)$ (C) $\left(-\frac{1}{a}, \infty\right)$

(D) $\left(\frac{1}{a}, \infty\right)$ (E) $(-\infty, -1)$

**56. Multiple Choice** If $f(0) = f'(0) = f''(0) = 0$, which of the following *must be true*?

(A) There is a local maximum of $f$ at the origin.

(B) There is a local minimum of $f$ at the origin.

(C) There is no local extremum of $f$ at the origin.

(D) There is a point of inflection of the graph of $f$ at the origin.

(E) There is a horizontal tangent to the graph of $f$ at the origin.

**57. Multiple Choice** The $x$-coordinates of the points of inflection of the graph of $y = x^5 - 5x^4 + 3x + 7$ are

(A) 0 only (B) 1 only (C) 3 only (D) 0 and 3 (E) 0 and 1

**58. Multiple Choice** Which of the following conditions would enable you to conclude that the graph of $f$ has a point of inflection at $x = c$?

(A) There is a local maximum of $f'$ at $x = c$.

(B) $f''(c) = 0$.

(C) $f''(c)$ does not exist.

(D) The sign of $f'$ changes at $x = c$.

(E) $f$ is a cubic polynomial and $c = 0$.

## Exploration

**59.** ***Graphs of Cubics*** There is almost no leeway in the locations of the inflection point and the extrema of $f(x) = ax^3 + bx^2 + cx + d$, $a \neq 0$, because the one inflection point occurs at $x = -b/(3a)$ and the extrema, if any, must be located symmetrically about this value of $x$. Check this out by examining **(a)** the cubic in Exercise 7 and **(b)** the cubic in Exercise 2. Then **(c)** prove the general case.

## Extending the Ideas

In Exercise 60, feel free to use a CAS (computer algebra system), if you have one, to solve the problem.

**60.** ***Quartic Polynomial Functions*** Let $f(x) = ax^4 + bx^3 + cx^2 + dx + e$ with $a \neq 0$.

(a) Show that the graph of $f$ has 0 or 2 points of inflection.

(b) Write a condition that must be satisfied by the coefficients if the graph of $f$ has 0 or 2 points of inflection.