Analysis of Curves worksheet

(1) $f(x) = x^3 + 3x^2 - 13x - 15$

(a) $f'(x) = 3x^2 + 6x - 13$ ◄ $f''(x) = 6x + 6 = 6(x+1)$ ◄

(b) Possible extrema, $f'(x) = 0$, $3x^2 + 6x - 13 = 0$

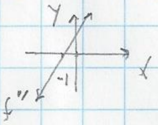
$$x = \frac{-6 \pm \sqrt{6^2 - 4(3)(-13)}}{2(3)} = \frac{-6 \pm \sqrt{192}}{6} \qquad x = -3.309 \quad x = 1.309$$

$f(-3.309) = 24.63$ $f''(-3.309) = -13.86 < 0 \Rightarrow (-3.309, 24.63)$ is rel. max ◄

$f(1.309) = -24.63$ $f''(1.309) = 13.86 > 0 \Rightarrow (1.309, -24.63)$ is rel. min ◄

(c) Possible inflection points, $f''(x) = 0$, $x + 1 = 0$, $x = -1$

$f(-1) = 0$



f'' switches signs at $x = -1 \Rightarrow (-1, 0)$ is inflection point ◄

(d) $f''(x) > 0$ for $x \in (-1, \infty)$ concave up ◄

$f''(x) < 0$ for $x \in (-\infty, -1)$ concave down ◄

(2) $f(x) = (x+3)(x-2)^3 + 80$

(a) $f'(x) = 1 \cdot (x-2)^3 + (x+3) \cdot 3(x-2)^2 = (x-2)^2 \left[(x-2) + 3(x+3) \right] =$

$= (x - 2 + 3x + 9)(x-2)^2 = (4x+7)(x-2)^2$ ◄

$f''(x) = 4 \cdot (x-2)^2 + (4x+7) \cdot 2(x-2) = 2(x-2)\left[2(x-2) + (4x+7)\right] =$

$= 2(2x - 4 + 4x + 7)(x-2) = 2(6x+3)(x-2) = 6(2x+1)(x-2)$ ◄

(b) Possible extrema, $f'(x) = 0$ $(4x+7)(x-2) = 0$ $x = -\frac{7}{4} = -1.75$, $x = 2$

$f(-1.75) = 14.08$ $f''(-1.75) = 56.25 > 0 \Rightarrow (-1.75, 14.08)$ is rel. min ◄

In fact, it is an absolute min.

$f(2) = 80$, $f''(2) = 0$ need to look at surrounding values

$f''(1.9) = -2.98 < 0$ concave down

$f''(2.1) = 3.12 > 0$ concave up

2 $\Rightarrow (2, 80)$ is inflection point.

(c) possible inflection points, $f''(x) = 0$, $(2x+1)(x-2) = 0$,

$x = -0.5$ $f(-0.5) = 40.94$ $f''(-0.6) = 3.12$ concave up

$f''(-0.4) = -2.88$ concave down $\Big\} \Rightarrow$

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The two inflection points are $(-0.5, 40.94)$ ◄ and $(2, 80)$ ◄

(d) $f''(x) > 0$ on $x \in (-\infty, -0.5) \cup (2, \infty)$ concave up ◄

$f''(x) < 0$ on $x \in (-0.5, 2)$ concave down ◄

(3) $f(x) = (x+3)(x-2)^4 + 40$

(a) $f'(x) = (x-2)^4 + (x+3)\cdot 4(x-2)^3 = (x-2)^3\left[(x-2) + 4(x+3)\right] =$

$= (x-2)^3(x-2+4x+12) = (5x+10)(x-2)^3 = 5(x+2)(x-2)^3$ ◄

$f''(x) = 5\left[(x-2)^3 + (x+2)\cdot 3(x-2)^2\right] = 5(x-2)^2\left[(x-2) + 3(x+2)\right] =$

$= 5(x-2+3x+6)(x-2)^2 = 5(4x+4)(x-2)^2 = 20(x+1)(x-2)^2$ ◄

(b) Possible extrema, $f'(x) = 0$, $(x+2)(x-2)^3 = 0$, $x = -2$, $x = 2$

$f(-2) = 296$, $f''(-2) = -320$ concave down $\Rightarrow (-2, 296)$ is rel. max ◄

$f(2) = 40$, $f''(2) = 0$ look at surrounding values

$\left.\begin{array}{l} f''(1.9) = 0.58 > 0 \\ f''(2.1) = 0.62 > 0 \end{array}\right\}$ both concave down

2

$(2, 40)$ is rel. min ◄
and is not an inflection point

(c) Possible inflection points, $f''(x) = 0$, $(x+1)(x-2)^2 = 0$

$x = -1$ & $x = 2$ (already looked at this), $f(-1) = 202$, look at surrounding values

$\left.\begin{array}{ll} f''(-1.1) = -19.22 < 0 & \text{concave down} \\ f''(-0.9) = 16.82 > 0 & \text{concave up} \end{array}\right\} \Rightarrow (-1, 202)$ is inflection point ◄

(d) $f''(x) > 0$ on $x \in (-1, 2) \cup (2, \infty)$ concave up ◄

Note: $x = 2$ possesses no concavity

$f''(x) < 0$ on $x \in (-\infty, -1)$ concave down ◄