HW # 7

5.1/5.3. Analysis of Curves

pg. 220

(3) $y = 2x^4 - 4x^2 + 1$, $y' = 8x^3 - 8x = 8x(x^2-1) = 8x(x+1)(x-1)$, $f(x)$

$y'' = 24x^2 - 8 = 8(3x^2-1)$, $y' = 0 \Rightarrow x(x+1)(x-1) = 0$, $x = -1, 0, 1$

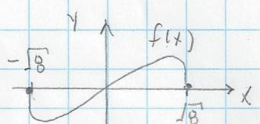
$y(-1) = -1$, $y''(-1) = 16 \Rightarrow (-1,-1)$ is rel min

$y(0) = 1$, $y''(0) = -8 \Rightarrow (0,1)$ is rel max — absolute

$y(1) = -1$, $y''(1) = 16 \Rightarrow (1,-1)$ is rel min

(5) $y = x\sqrt{8-x^2}$, $y' = \sqrt{8-x^2} + x \cdot \frac{1}{2}(8-x^2)^{-1/2} \cdot -2x = \dfrac{8-x^2-x^2}{\sqrt{8-x^2}} = \dfrac{2(4-x^2)}{\sqrt{8-x^2}} = \dfrac{u}{v}$

$$y'' = \frac{u'v-uv'}{v^2} = \frac{\dfrac{2\cdot -2x(8-x^2)}{\sqrt{8-x^2}} - 2(4-x^2)\cdot\frac{1}{2}\frac{1}{\sqrt{8-x^2}}\cdot -2x}{8-x^2} = \frac{2x(x^2-12)}{(\sqrt{8-x^2})^3}$$



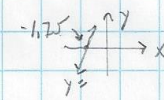
$\sqrt{8} = 2.828$ $y' = 0 \Rightarrow 4 - x^2 = 0$, $x^2 = 4$, $x = \pm 2$

$f(-2) = -4$ $f''(-2) = 4 \Rightarrow (-2,-4)$ is rel min — absolute

$f(2) = 4$ $f''(2) = -4 \Rightarrow (2, 4)$ is rel max

Endpoint values: $(-\sqrt{8}, 0)$ is rel max $(\sqrt{8}, 0)$ is rel min

(7) $y = 4x^3 + 21x^2 + 36x - 20$, $y' = 12x^2 + 42x + 36$, $y'' = 24x + 42 = 6(4x+7) = 0$, $x = -1.75$



$y(-1.75) = -40.125 \Rightarrow (-1.75, -40.125)$ is inflection point

(y'' switches signs at $x = -1.75$)

$y''(x) < 0$ on $x \in (-\infty, -1.75)$ concave down

$y''(x) > 0$ on $x \in (-1.75, \infty)$ concave up

(19) $y = \dfrac{x^3 - 2x^2 + x - 1}{x-2} = \dfrac{u}{v}$, $y' = \dfrac{u'v - uv'}{v^2} = \dfrac{(3x^2-4x+1)(x-2) - (x^3-2x^2+x-1)(1)}{(x-2)^2} =$

$= \dfrac{2x^3 - 8x^2 + 8x - 1}{(x-2)^2} = \dfrac{u}{v}$,

$y'' = \dfrac{u'v - uv'}{v^2} = \dfrac{(6x^2-16x+8)(x-2)^2 - (2x^3-8x^2+8x-1)\cdot 2(x-2)}{(x-2)^{\cancel{4}\,3}} =$

$= \dfrac{2(x^3 - 6x^2 + 12x - 7)}{(x-2)^3} = \dfrac{2(x-1)(x^2-5x+7)}{(x-2)^3}$

1	1	-6	12	-7
		1	-5	7
	1	-5	7	0

$y''(x) = (x-1)(x^2-5x+7) = 0$ $\quad b^2-4ac = 25-4(1)(7) = -3$ (two complex roots) and $x=1$

$y(1) = 1 \quad y''(0.9) = 0.497 \quad y''(1.1) = -0.743$ so $(1,1)$ is inflection point

(20) $y = \dfrac{x}{x^2+1} = \dfrac{u}{v} \qquad y' = \dfrac{u'v-uv'}{v^2} = \dfrac{1(x^2+1)-x\cdot 2x}{(x^2+1)^2} = \dfrac{1-x^2}{(x^2+1)^2} = \dfrac{u}{v}$

$$y'' = \frac{u'v-uv'}{v^2} = \frac{-2x(x^2+1)^{\cancel{2}} - (1-x^2)2\cancel{(x^2+1)}\cdot 2x}{(x^2+1)^{\cancel{4}\,3}} = \frac{2x(x^2-3)}{(x^2+1)^3}$$

$y''=0 \Rightarrow x=0,\ x=\pm\sqrt{3} = \pm 1.732$

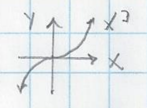
$y(-\sqrt{3}) = -0.433 \quad y''(-1.8) = -0.0113 \quad y''(-1.7) = 0.0064$ ok

$y(0) = 0 \quad y''(-0.1) = 0.5804 \quad y''(0.1) = -0.5804$ ok

$y(\sqrt{3}) = 0.433 \quad y''(1.7) = -0.0064 \quad y''(1.8) = 0.0113$ ok

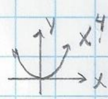
So, inflection points are $(-1.732, -0.433)$, $(0,0)$, $(1.732, 0.433)$

(41) Look at $f(x) = x^3$



$f'(x) = 3x^2$ is zero at $x=0$, but it is not an extremum.

(42) Look at $f(x) = x^4$

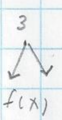


$f''(x) = 12x^2$ is zero at $x=0$, but it is not an inflection point.

Supplemental:

(1) $f(x) = \begin{cases} x^2-4x+6, & 0 \le x \le 3 \\ x^2-10x+24, & 3 < x \le 6 \end{cases}$ $\quad f(3^-) = 3,\ f(3^+) = 3$ so is continuous at $x=3$

(a) Left: $f'(x) = 2x-4 = 2(x-2)$ $\quad f'(3^-) = 2$
Right $f'(x) = 2x-10 = 2(x-5)$ $\quad f'(3^+) = -4$



(a kink)
$(3,3)$ is rel. max

$f'(x) = \begin{cases} 2(x-2), & 0 \le x < 3 \\ 2(x-5), & 3 < x \le 6 \end{cases}$ $\quad f''(x) = \begin{cases} 2, & 0 \le x < 3 \text{ concave up} \\ 2, & 3 < x \le 6 \text{ concave up} \end{cases}$

(b) $f'(x) = 0 \quad (x-2) = 0 \quad x=2 \quad f(2) = 2 \quad f''(2) > 0 \quad (2,2)$ is rel. min
$(x-5) = 0 \quad x=5 \quad f(5) = -1 \quad f''(5) > 0 \quad (5,-1)$ is rel min

Endpoints $x=0 \quad f(0) = 6 \quad f'(0) = -4$ $\quad$ -4 f(x) $\quad (0,6)$ is rel max
$x=6 \quad f(6) = 0 \quad f'(6) = 2$ $\quad$ 6 f(x) $\quad (6,0)$ is rel max

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(0,6) abs. max ◂ (2,2) rel min ◂
(3,3) rel. max ◂ (5,−1) abs. min ◂
(6,0) rel. max ◂

5.1/5.3. Absolute and Relative Extrema

pg. 200

(1) (0,2) is abs max ◂ (−2,0) and (2,0) are abs min ◂

(3) (0,5) is abs. max ◂ Note that (2,0) is not a min because it is not on the graph.

(5) $x = a$ rel. min ◂ $x = c_1$ rel max ◂
$x = c_2$ abs. min ◂ $x = b$ abs max ◂

(11) $f(x) = \frac{1}{x} + \ln x$, $x \in [0.5, 4]$, $f'(x) = -\frac{1}{x^2} + \frac{1}{x}$, $f''(x) = \frac{2}{x^3} - \frac{1}{x^2}$

$f'(x) = \frac{x-1}{x^2} = 0 \quad x = 1$, $f(1) = 1$, $f''(1) = 1$ ⤻ (1,1) is rel. min

endpoints... $f(0.5) = 1.307 \quad f'(0.5) = -2$ 0.5 ↘ f(x) (0.5, 1.307) is rel max

$f(4) = 1.636 \quad f'(4) = 0.1875$ ↙ 4 f(x) (4, 1.636) is rel max

(0.5, 1.307) rel max ◂
(4, 1.636) abs max ◂ (1,1) abs min ◂

(21) $y = x^3 + x^2 - 8x + 5$, $y' = 3x^2 + 2x - 8$, $y'' = 6x + 2 = 2(3x+1)$

$ac = (3)(-8) = -24 = -2^3 \cdot 3 = tu$, $b = 2 = t + u$, $t = -4, u = 6$

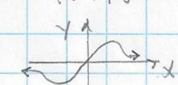
$y' = 3x^2 - 4x + 6x - 8 = x(3x-4) + 2(3x-4) = (x+2)(3x-4) = 0 \quad x = -2, \frac{4}{3}$

$f(-2) = 17$, $f''(-2) = -10$ ⤺ (−2, 17) is rel max ◂

$f(\frac{4}{3}) = -\frac{41}{27}$, $f''(\frac{4}{3}) = 10$ ⤻ $(1\frac{1}{3}, -1\frac{14}{27})$ is rel min ◂

(29) From pg. 220, problem 20 $y = \frac{x}{x^2+1}$, $y' = \frac{1-x^2}{(x^2+1)^2}$, $y'' = \frac{2x(x^2-3)}{(x^2+1)^3}$

$y'(x) = 0$, $1 - x^2 = 0$, $x^2 = 1$, $x = \pm 1$

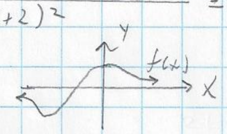


$y(-1) = -0.5 \quad y''(-1) = 0.5$ ⤻ (−1, −0.5) rel. min ◂
$y(1) = 0.5 \quad y''(1) = -0.5$ ⤺ (1, 0.5) rel. max ◂

absolute

(30) $y = \dfrac{x+1}{x^2+2x+2} = \dfrac{u}{v}$, $y' = \dfrac{u'v-uv'}{v^2} = \dfrac{1(x^2+2x+2)-(x+1)(2x+2)}{(x^2+2x+2)^2} =$

$= \dfrac{x^2+2x+2-2(x+1)^2}{(x^2+2x+2)^2} = \dfrac{-x^2-2x}{(x^2+2x+2)^2} = \dfrac{u}{v}$



$y'' = \dfrac{u'v-uv'}{v^2} = \dfrac{-2(x+1)(x^2+2x+2)^{\cancel{2}} - (-x^2-2x)\cdot 2(x^2+2x+2)(2x+2)}{(x^2+2x+2)^4} =$

$= \dfrac{2(x+1)(x^2+2x-2)}{(x^2+2x+2)^3}$ $y'(x) = 0 = -x^2-2x$, $x(x+2)=0$, $x=-2, 0$

$y(0) = 0.5$ $y''(0) = -0.5$ ↶ $(0, 0.5)$ rel max ← absolute

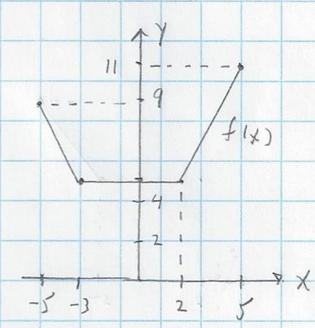
$y(-2) = -0.5$ $y''(-2) = 0.5$ ↷ $(-2, -0.5)$ rel min

(31) $f(x) = |x-2| + |x+3|$, $x \in [-5, 5]$

$(-5, 9)$ rel max

$(5, 11)$ abs max

$(x, 5)$, $x \in [-3, 2]$ abs min

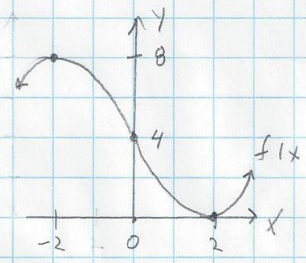


pg. 220 5.3. Using f' and f'' to Sketch f

(45) $f(-2)=8$ $f(0)=4$ $f(2)=0$

	$x \in (-\infty,-2)$	$x=-2$	$x \in (-2,2)$	$x=2$	$x \in (2,\infty)$
f'	+	0	−	0	+

	$x \in (-\infty, 0)$	$x \in (0, \infty)$
f''	− ↶	+ ↷



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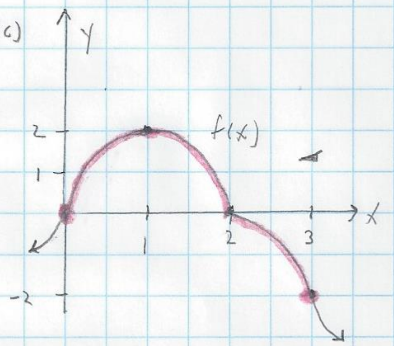
(49)

	$x=0$	$x\in(0,1)$	$x=1$	$x\in(1,2)$	$x=2$	$x\in(2,3)$	$x=3$
f	0	+	2	+	0	−	−2
f'	3	+	0	−	D.N.E.	−	−3
f''	0	−	−1	−	D.N.E.	−	0

(c)



(a) $(1,2)$ abs. max

$(3,-2)$ abs. min

(b) no inflections due to the closed interval