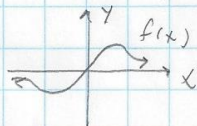


Quiz #7 Study Guide

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$$(1) f(x) = \frac{x}{x^2+4} = \frac{u}{v} \quad (a) f'(x) = \frac{u'v - uv'}{v^2} = \frac{1 \cdot (x^2+4) - x \cdot 2x}{(x^2+4)^2} = \frac{-x^2+4}{(x^2+4)^2} = \frac{4}{v} -$$

$$f''(x) = \frac{u'v - uv'}{v^2} = \frac{-2x(x^2+4)^2 - (-x^2+4)2(x^2+4)2x}{(x^2+4)^4} = \frac{2x(x^2-12)}{(x^2+4)^3}$$

(b)  $f'(x) = 0, -x^2+4=0, x^2=4, x=-2, 2$

$f(-2) = -0.25, f''(-2) = 0.0625 \rightarrow$ so min $\Rightarrow (-2, -0.25)$ is abs min \leftarrow
 $f(2) = 0.25, f''(2) = -0.0625 \rightarrow$ so max $\Rightarrow (2, 0.25)$ is abs max \leftarrow

(c) $f''(x) = 0 \Rightarrow x(x^2-12) = 0 \Rightarrow x=0, x^2=12 \Rightarrow x = \pm\sqrt{12} = \pm 2\sqrt{3} = \pm 3.464$

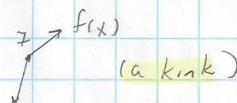
$f(-3.464) = -0.2165, f(0) = 0, f(3.464) = 0.2165 \Rightarrow$

$(-3.464, -0.2165) \leftarrow (0, 0) \leftarrow (3.464, 0.2165) \leftarrow$ are the inflection points.

(2) $f(x) = \begin{cases} \frac{1}{2}x^3 - \frac{49}{8}x^2 + 21x - \frac{107}{8}, & 0 \leq x \leq 7 \\ -\frac{5}{8}x^2 + \frac{23}{2}x - \frac{359}{8}, & 7 < x \leq 12 \end{cases} \quad (a) \left. \begin{matrix} f(7^-) = 5 \\ f(7^+) = 5 \end{matrix} \right\} \Rightarrow$ so is continuous at $x=7 \leftarrow$

Left: $f'(x) = \frac{3}{2}x^2 - \frac{49}{4}x + 21, f'(7^-) = 8.75$

Right: $f'(x) = -\frac{5}{4}x + \frac{23}{2}, f'(7^+) = 2.75$



(b) so not differentiable at $x=7 \leftarrow$

(c) No \leftarrow

(d) $f'(x) = \begin{cases} \frac{3}{2}x^2 - \frac{49}{4}x + 21, & 0 \leq x \leq 7 \\ -\frac{5}{4}x + \frac{23}{2}, & 7 < x \leq 12 \end{cases} \quad f''(x) = \begin{cases} 3x - \frac{49}{4}, & 0 \leq x \leq 7 \\ -\frac{5}{4}, & 7 < x \leq 12 \end{cases} \leftarrow$

(e) $f'(x) = 0 \Rightarrow \frac{3}{2}x^2 - \frac{49}{4}x + 21 = 0 \Rightarrow x = \frac{\frac{49}{4} \pm \sqrt{(\frac{49}{4})^2 - 4(\frac{3}{2})(21)}}{2(\frac{3}{2})} \Rightarrow x = 2.4482, x = 5.7185$

$f(2.4482) = 8.6628, f''(2.4482) = -4.905 \rightarrow$ max

$f(5.7185) = -0.0806, f''(5.7185) = 4.905 \rightarrow$ min

$f'(x) = 0 \Rightarrow -\frac{5}{4}x + \frac{23}{2}, x = 9.2, f(9.2) = 8.025, f'' = -\frac{5}{4} \rightarrow$ max

Endpoints: $f(0) = -13.375, f'(0) = 21 \rightarrow$ min

$f(12) = 3.125, f'(12) = -2.5 \rightarrow$ min

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$(2.4482, 8.6628)$ abs max \leftarrow

$(0, -13.375)$ abs min \leftarrow

$(9.2, 8.025)$ rel max \leftarrow

$(5.7185, -0.0806)$ rel min \leftarrow

$(12, 3.125)$ rel min \leftarrow

(f) $f''(x) = 0$, $3x - \frac{49}{4} = 0$, $x = \frac{49}{12} = 4.08\bar{3}$, $f(4.0833) = 4.2911$

$(4.0833, 4.2911)$ is inflection point \leftarrow

$f''(7^-) = 8.75$, $f''(7^+) = -1.25$.

sign switch $\Rightarrow (7, 5) \leftarrow$

