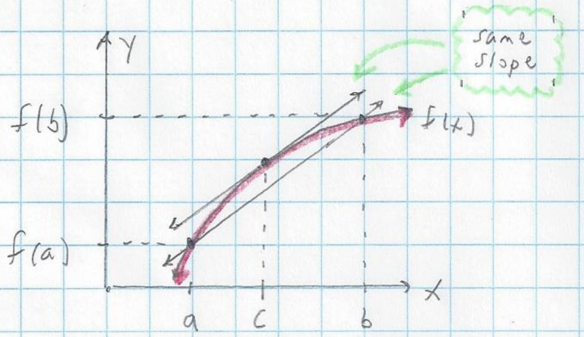
5.2. Mean Value Theorem

Mean Value Theorem (MVT)

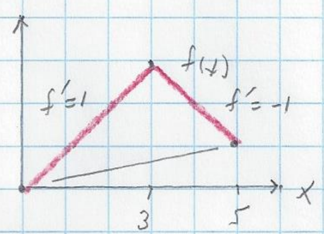


If $y = f(x)$ is differentiable on $x \in [a, b]$, then there is at least one value $c \in [a, b]$ such that $f'(c)$ is equal to the average slope of f over $x \in [a, b]$, i.e.,

$$f'(c) = \frac{f(b) - f(a)}{b - a}.$$

Example #1. Does not satisfy the MVT.

Look at $f(x) = -|x-3|$ on $x \in [0, 5]$. $f(x)$ is not differentiable at $x = 3$, so the MVT does not hold.



$$\text{average slope} = \frac{f(5) - f(0)}{5 - 0} = \frac{1 - 0}{5 - 0} = \frac{1}{5}$$

Example #2. For $f(x) = -x^2 + 8x - 7$ on $x \in [1, 5]$, find the value of $c \in [1, 5]$ which is guaranteed to exist by the MVT.

SOLUTION:

$$\frac{f(5) - f(1)}{5 - 1} = \frac{8 - 0}{5 - 1} = 2 \quad f'(x) = -2x + 8, \quad f'(c) = 2 = -2c + 8, \quad 2c = 6, \quad c = 3$$

CLASS WORK

For $f(x) = x^2 - 8x + 17$ on $x \in [1, 6]$, find the value of $c \in [1, 6]$ which is guaranteed to exist by the MVT.

SOLUTION

$$\frac{f(6) - f(1)}{6 - 1} = \frac{5 - 10}{6 - 1} = -1 \quad f'(x) = 2x - 8 \quad -1 = 2c - 8 \quad 2c = 7 \quad c = \frac{7}{2} = 3.5$$