

Quiz #9 Study Guide

10F1

$$(1) f(x) = x^3 - 6x^2 - 13x + 42, x \in [-4, 10] \quad \frac{f(10) - f(-4)}{10 - (-4)} = 27$$

$$f'(x) = 3x^2 - 12x - 13, \quad 30^2 - 12C - 13 = 27, \quad 30^2 - 12C - 40 = 0$$

$$C = \frac{12 \pm \sqrt{12^2 - 4(3)(-40)}}{2(3)} = \frac{12 \pm \sqrt{624}}{6} \quad C = -2.163 \leftarrow$$

$$C = 6.163 \leftarrow$$

$$(2) A = 2xy \quad 2x + 3y = 200 \quad 3y = 200 - 2x \quad y = \frac{200}{3} - \frac{2}{3}x \quad A = 2x \left(\frac{200}{3} - \frac{2}{3}x \right) =$$

$$= -\frac{4}{3}x^2 + \frac{400}{3}x, \quad A'(x) = -\frac{8}{3}x + \frac{400}{3} = 0 \quad x = 50 \text{ ft} \quad y = 33\frac{1}{3} \text{ ft}$$

$$A = 3333\frac{1}{3} \text{ ft}^2 \quad A'' = -\frac{8}{3} \leftarrow \text{so max}$$

$$(3) V = 1000 = x^2y \quad y = \frac{1000}{x^2} \quad S = x^2 + 4xy = x^2 + 4x \cdot \frac{1000}{x^2} = x^2 + \frac{4000}{x}$$

$$S'(x) = 2x - \frac{4000}{x^2} = 0, \quad x^3 = 2000, \quad x = 12.60 \text{ cm} \quad y = 6.30 \text{ cm}$$

$$S = 476.22 \text{ cm}^2 \quad S''(x) = 2 + \frac{8000}{x^3}, \quad S''(12.60) = 6 \leftarrow \text{so min}$$

$$(4) f(x) = \sqrt{x} \quad f'(x) = \frac{1}{2\sqrt{x}} \quad f(100) = 10 \quad f'(100) = \frac{1}{20}$$

$$L(x) = \frac{1}{20}(x - 100) + 10, \quad L(99) = 9.95 \leftarrow \text{error} = \frac{L(99) - f(99)}{f(99)} = 1.263 \times 10^{-3} \%$$

$$(5) V = s^3 \quad s = 10 \quad ds = 0.1 \quad dV = 3s^2 ds = 3(10)^2(0.1) = 30 \leftarrow$$

$$\Delta V = (10.1)^3 - 10^3 = 30.301 \quad \text{error} = \frac{dV - \Delta V}{\Delta V} = -0.993\% \leftarrow$$

$$(6) \sin \theta = \frac{h}{r} \quad h = r \sin \theta \quad A = \frac{1}{2}bh = \frac{1}{2}(6)r \sin \theta = 3r \sin \theta$$

$$\frac{dA}{dt} = 15 \cos \theta \frac{d\theta}{dt}, \quad x^2 = r^2 + 6^2 - 2(r)(6) \cos \theta, \quad x^2 = 61 - 60 \cos \theta$$

$$2x \frac{dx}{dt} = -60 \sin \theta \frac{d\theta}{dt} = -60 \sin \theta \frac{d\theta}{dt} \quad \frac{d\theta}{dt} = \frac{x}{30 \sin \theta} \frac{dx}{dt}$$

$$\frac{dA}{dt} = 15 \cos \theta \cdot \frac{x}{30 \sin \theta} \frac{dx}{dt} = \frac{x}{2 \tan \theta} \frac{dx}{dt} \quad x^2 = 61 - 60 \cos \theta \quad -12 = -60 \cos \theta$$

$$\cos \theta = \frac{1}{5} \quad \theta = 78.46^\circ \quad \frac{dA}{dt} = \frac{7}{2 \tan 78.46^\circ} (0.2) = 0.143 \frac{\text{m}^2}{\text{sec}} \leftarrow$$