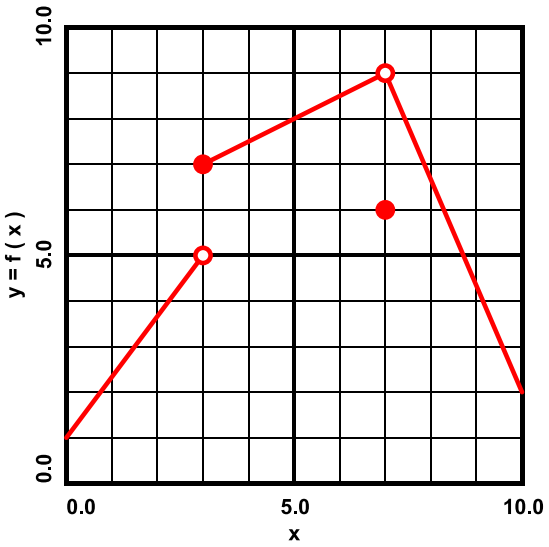
# AP CALCULUS AB

# FALL FINAL STUDY GUIDE

Problems **1** through **3** concern the graph of $y = f(x)$.




**1)** $\lim_{x \to 7^-} f(x) = ?$

**2)** $\lim_{x \to 7^+} f(x) = ?$

**3)** $\lim_{x \to 7} f(x) = ?$

**4)**

$$\lim_{x \to 0} \frac{1}{x^3} = ?$$

**5)**

$$\lim_{x \to 0} \frac{1}{|x|} = ?$$

**6)**

$$\lim_{x \to 0} x^2 \cos\left(\frac{2\pi}{x}\right) = ?$$

**7)**

$$\lim_{x \to \infty} \frac{x^{10}}{\log_9 x} = ?$$

Problems **8** and **9** concern the function

$$f(x) = \frac{12x^2 + 17}{3x^2 + x - 10}.$$

**8)** Find the horizontal asymptote of $y = f(x)$.

**9)** Find the vertical asymptotes of $y = f(x)$.

**10)** Find the end-behavior function $E(x)$ of

$$f(x) = \frac{3x^3 + 13x^2 - 6x - 40}{x - 3}.$$

**11)** Is

$$f(x) = \frac{x + 8}{|x + 8|}$$

differentiable at $x = -8$?

**12)** Is

$$f(x) = \begin{Bmatrix} -x^2 + 6x & , & -\infty < x \leq 4 \\ x^2 - 10x + 32 & , & 4 < x < \infty \end{Bmatrix}$$

differentiable at $x = 4$?

**13)** For $f(x) = -x^2 + 6x$ defined on $x \in [0\,,5]$, find the value of $c \in (0\,,5)$ such that $f(c) = 4$ which is guaranteed to exist via the Intermediate Value Theorem for Continuous Functions.

**14)** Calculate the average rate of change of $f$ with respect to $x$ of $f(x) = 4x^2 - 18x + 105$ over $x \in [2\,,8]$.

**15)** Calculate $\frac{f(x+h)-f(x)}{h}$ for $f(x) = \frac{1}{x^4}$.

**16)** Calculate

$$\lim_{h \to 0} \frac{(3 + h)^5 - 243}{h}.$$

Problems **17** and **18** concern the function $f(x) = -x^2 + 8x - 7$.

**17)** Find the equation of the line tangent to $y = f(x)$ at $x = 5$.

**18)** Find the equation of the line normal to $y = f(x)$ at $x = 5$.

**19)** Calculate $f'(x)$ for $f(x) = 7x^7 + 5x^5 + 3x^3 + x$.

**20)** Calculate $f'(x)$ for

$$f(x) = 11 \cdot \sqrt[5]{x^7} + \frac{13}{\sqrt[9]{x^2}}.$$

**21)** Given $y = f(x)$ defined by the table

| $x$ | 1 | 4 | 8 |
|---|---|---|---|
| $f(x)$ | 11 | 48 | 112 |

estimate $f'(6)$.

**22)** Given $f(x) = x^2 + 7x - 9$ defined on $x \in [2\,,9]$, calculate

$$\lim_{x \to 2^+} \frac{f(x) - f(2)}{x - 2}.$$

**23)** Find the $x$ – value for which $f(x) = 5x^2 + 32x - 21$ has a horizontal tangent.

**24)** Calculate $f'(x)$ for $f(x) = x^5 \tan x$.

**25)** Calculate $f'(x)$ for

$$f(x) = \frac{\cos x}{x^3 - 8}.$$

Problems **26** and **27** concern Mitchell, who hits a home run. When the baseball is hit, it has an initial upward velocity of 115 ft/sec. The ball then lands in the stands, which are 80 ft above home plate. Use $g = 32$ ft/sec$^2$.

**26)** Find the maximum height of the ball.

**27)** Find the speed of the ball when it lands in the stands.

**28)** Calculate $f'(x)$ for $f(x) = \csc(5x^2 + 7)$.

**29)** Calculate $f'(x)$ for $f(x) = \ln|\cos(7x - 9)|$.

**30)** For $f(x) = x^3 - 4x^2 - 11x + 30$, calculate

$$\left.\frac{\mathrm{d}f^{-1}}{\mathrm{d}x}\right|_{(28\,,-2\,)}.$$

**31)** Find the slope of the curve $x^3y + xy^3 - 300 = 0$ at $(x, y) = (3\,,4)$.

**32)** Calculate $f'(x)$ for $f(x) = \csc^{-1}\left(\sqrt{x^5}\right)$ defined on $x \in [1, \infty)$.

**33)** Calculate $f'(x)$ for $f(x) = 5^x \cos x$.

**34)** Calculate $f'(x)$ for $f(x) = x^4 \log_5 x$.

Problems **35** through **37** concern the function $f(x) = x^3 + 6x^2 - 15x + 19$.

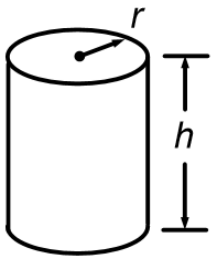
**35)** Find the $x$ – value of the relative maximum.

**36)** Find the $x$ – value of the relative minimum.

**37)** Find the $x$ – value of the inflection point.

**38)** For $f(x) = x^3 + x^2 - 17x + 15$ defined on $x \in [-6\,,5]$, find the values of $c \in [-6\,,5]$ which are guaranteed to exist by the Mean Value Theorem.

**39)** The open-topped, cylindrical can shown has volume $V = \pi r^2 h = 43.3$ in$^3$ (1 pint) and surface area $S = \pi r^2 + 2\pi rh$.




Find the value of $h$ which minimizes its surface area.

**40)** The hot-air balloon shown has vertical velocity $\frac{\mathrm{d}h}{\mathrm{d}t} = 15$ ft/sec, and is being observed on the ground from a distance of $x = 100$ ft. When $h = 400$ ft, calculate $\frac{\mathrm{d}\theta}{\mathrm{d}t}$ in degrees/sec.