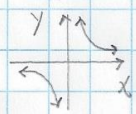
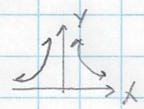
Fall Final Study Guide

(1) $\lim_{x \to 7^-} f(x) = 9$ (2) $\lim_{x \to 7^+} f(x) = 9$

(3) $\lim_{x \to 7} f(x) = 9 = \lim_{x \to 7^-} f(x) = \lim_{x \to 7^+} f(x)$, even though $f(7) = 6$

(4) 

$\lim_{x \to 0} \frac{1}{x^3}$ does not exist because $\lim_{x \to 0^-} \frac{1}{x^3} = -\infty \neq \lim_{x \to 0^+} \frac{1}{x^3} = \infty$

(5) 

$\lim_{x \to 0} \frac{1}{|x|} = \infty$ because $\lim_{x \to 0^-} \frac{1}{|x|} = \lim_{x \to 0^+} \frac{1}{|x|} = \infty$

(6) $\lim_{x \to 0} x^2 \cos\left(\frac{2\pi}{x}\right) = 0 \cdot \left[\begin{matrix}\text{bounded between} \\ -1 \text{ and } 1\end{matrix}\right] = 0$

(7) $\lim_{x \to \infty} \frac{x^{10}}{\log_9 x} = \frac{\infty}{\infty} = \infty$ ← Dominates

(9) $ac = 3 \cdot -10 = -2 \cdot 3 \cdot 5 = -30 = tu$ $b = 1 = t + u$ $t = 6$ $u = -5$

$3x^2 + 6x - 5x - 10 = 3x(x+2) - 5(x+2) = (3x-5)(x+2) \Rightarrow$

V.A.'s are $x = -2$ and $x = \frac{5}{3}$

(8) As $x \to \pm\infty$, $f(x) \to \frac{12x^2}{3x^2} = 4 \Rightarrow y = 4$ is H.A.

(10)

3	3	13	−6	−40
		9	66	180
	3	22	60	140

$\Rightarrow E(x) = 3x^2 + 22x + 60$

(11) No because $f(x)$ has a jump at $x = -8$

(12) $f(4^-) = 8$, $f(4^+) = 8$, so continuous. Left: $f'(x) = -2x + 6$ Right: $f'(x) = 2x - 10$

$f'(4^-) = -2$, $f'(4^+) = -2$, so yes is differentiable at $x = 4$

(13) $f(0) = 0$, $f(5) = 5$ so IVTCF holds. $-c^2 + 6c = 4$, $c^2 - 6c + 4 = 0$

$c = \frac{6 \pm \sqrt{6^2 - 4(1)(4)}}{2(1)} = \frac{6 \pm \sqrt{20}}{2} = \frac{6 \pm 2\sqrt{5}}{2} = 3 \pm \sqrt{5}$

$c = 3 - \sqrt{5} = 0.764$ ✓

$c = 3 + \sqrt{5} = 5.236$ ✗

(14) $\dfrac{f(8)-f(2)}{8-2} = 22$

(15) $f(x+h)-f(x) = \dfrac{1}{(x+h)^4} - \dfrac{1}{x^4} = \dfrac{x^4}{(x+h)^4x^4} - \dfrac{(x+h)^4}{x^4(x+h)^4} = \dfrac{x^4-(x+h)^4}{(x+h)^4x^4} =$

$= \dfrac{x^4-(x^4+4x^3h+6x^2h^2+4xh^3+h^4)}{(x+h)^4x^4} = -\dfrac{4x^3h+6x^2h^2+4xh^3+h^4}{(x+h)^4x^4}$

$\dfrac{f(x+h)-f(x)}{h} = -\dfrac{4x^3+6x^2h+4xh^2+h^3}{(x+h)^4x^4}$

(16) $\lim\limits_{h\to 0}\dfrac{(3+h)^5-243}{h} = \lim\limits_{h\to 0}\dfrac{(3+h)^5-3^5}{h} = \left.\dfrac{dx^5}{dx}\right|_{x=3} = \left.5x^4\right|_{x=3} = 5\cdot 3^4 = 405$

(17) $f'(x) = -2x+8$, $f(5) = 8$, $f'(5) = -2$, $y = -2x+b$, $8 = -2(5)+b$, $b = 18$

$y = -2x+18$

(18) $y = \frac{1}{2}x + b$, $8 = \frac{1}{2}(5)+b$, $b = \frac{11}{2}$, $y = \frac{1}{2}x + \frac{11}{2}$

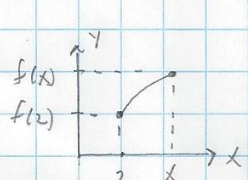
(19) $f'(x) = 49x^6 + 25x^4 + 9x^2 + 1$

(20) $f(x) = 11\cdot x^{7/5} + 13x^{-2/9}$, $f'(x) = 11\cdot\frac{7}{5}x^{7/5-1} + 13\left(-\frac{2}{9}\right)x^{-2/9-1} =$

$= \frac{77}{5}x^{2/5} - \frac{26}{9}x^{-11/9} = \dfrac{77\cdot\sqrt[5]{x^2}}{5} - \dfrac{26}{9\cdot\sqrt[9]{x^{11}}}$

(21) $\frac{1}{2}(4+8) = 6$, $f'(6) \approx \dfrac{112-48}{8-4} = 16$

(22)



$\lim\limits_{x\to 2^+}\dfrac{f(x)-f(2)}{x-2} = \left.\dfrac{df}{dx}\right|_{x=2} = \left.(2x+7)\right|_{x=2} = 11$

(23) $f'(x) = 10x+32 = 0 \Rightarrow x = -3.2$

(24) $f(x) = x^5\tan x = uv$, $f'(x) = u'v+uv' = 5x^4\tan x + x^5\sec^2 x$

(25) $f(x) = \dfrac{\cos x}{x^3-8} = \dfrac{u}{v}$, $f'(x) = \dfrac{u'v-uv'}{v^2} = \dfrac{-\sin x(x^3-8) - \cos x\cdot 3x^2}{(x^3-8)^2} =$

$= -\dfrac{(x^3-8)\sin x + 3x^2\cos x}{(x^3-8)^2}$

(26) $y = 115t - 16t^2 \quad v = \frac{dy}{dt} = 115 - 32t \quad v = 0 \Rightarrow t = 3.59375$

$y(3.59375) = 206.64\ ft$

(27) $80 = 115t - 16t^2, \quad 16t^2 - 115t + 80 = 0 \quad t = \frac{115 \pm \sqrt{115^2 - 4(16)(80)}}{2(16)} =$

$= \frac{115 \pm \sqrt{8105}}{32} \quad t = 0.780$ on way up $\quad t = 6.407 \quad v(6.407) = -90.03 \Rightarrow 90.03\ \frac{ft}{sec}$

(28) $f(x) = \csc(5x^2+7), \quad f'(x) = -\csc(5x^2+7)\cot(5x^2+7) \cdot 10x =$
$= -10x \csc(5x^2+7)\cot(5x^2+7)$

(29) $f(x) = \ln|\cos(7x-9)|, \quad f'(x) = \frac{1}{|\cos(7x-9)|} \cdot \frac{|\cos(7x-9)|}{\cos(7x-9)} \cdot -\sin(7x-9) \cdot 7 =$

$= -\frac{7\sin(7x-9)}{\cos(7x-9)} = -7\tan(7x+9)$

(30) Note: $f(-2) = 28, \quad f'(x) = 3x^2 - 8x - 11, \quad f'(-2) = 17 \Rightarrow \left.\frac{df^{-1}}{dx}\right|_{(28,-2)} = \frac{1}{17}$

(31) $x^3y + xy^3 - 300 = 0, \quad 3x^2y + x^3y' + 1 \cdot y^3 + x \cdot 3y^2y' = 0,$

$3x^2y + y^3 + y'(x^3 + 3xy^2) = 0 \quad y' = -\frac{3x^2y + y^3}{3xy^2 + x^3} = -\frac{3 \cdot 9 \cdot 4 + 64}{3 \cdot 3 \cdot 16 + 27} = -\frac{172}{171}$

(32) $f(x) = \csc^{-1} g \quad g = \sqrt{x^5} = x^{5/2}, \quad \frac{dg}{dx} = \frac{5}{2}x^{3/2} = \frac{5\sqrt{x^3}}{2}, \quad \frac{df}{dx} = \frac{df}{dg}\frac{dg}{dx} =$

$= \frac{-1}{|g|\sqrt{g^2-1}} \cdot \frac{5\sqrt{x^3}}{2} = \frac{-5\sqrt{x^3}}{2|\sqrt{x^5}|\sqrt{(\sqrt{x^5})^2 - 1}} = \frac{-5}{2x\sqrt{x^5-1}}$

(33) $f(x) = 5^x \cos x, \quad f'(x) = (\ln 5)5^x \cos x + 5^x \cdot -\sin x = [(\ln 5)\cos x - \sin x] \cdot 5^x$

(34) $f(x) = x^4 \log_5 x, \quad f'(x) = 4x^3 \log_5 x \frac{(\ln 5)}{(\ln 5)} + x^4 \cdot \frac{1}{(\ln 5)x} = \frac{x^3}{\ln 5} \cdot [4(\ln 5)\log_5 x + 1]$

(35) $f(x) = x^3 + 6x^2 - 15x + 19, \quad f'(x) = 3x^2 + 12x - 15 = 3(x^2+4x-5) = 3(x+5)(x-1)$

$f''(x) = 6x + 12 = 6(x+2)$

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$f'(x) = 0 \Rightarrow x = -5,\ x = 1$ $\quad f''(-5) < 0$ $\Rightarrow x = -5$ is rel max

(36) $f''(1) > 0$ $\Rightarrow x = 1$ is rel min

(37) $f''(x) = 0 \Rightarrow x = -2$ is inflection point

(38) $f(x) = x^3 + x^2 - 17x + 15$, $f'(x) = 3x^2 + 2x - 17$, $\dfrac{f(5) - f(-6)}{5 - -6} = 13$

$3c^2 + 2c - 17 = 13$, $3c^2 + 2c - 30 = 0$ $\quad c = \dfrac{-2 \pm \sqrt{2^2 - 4(3)(-30)}}{2(3)} = \dfrac{-2 \pm \sqrt{364}}{6}$

$c = -3.513$ $\quad c = 2.846$

(39) $\pi r^2 h = 43.3$, $h = \dfrac{43.3}{\pi r^2}$, $S = \pi r^2 + 2\pi r h = \pi r^2 + 2\pi r \cdot \dfrac{43.3}{\pi r^2} = \pi r^2 + \dfrac{86.6}{r}$

$S'(r) = 2\pi r - \dfrac{86.6}{r^2} = 0 \Rightarrow r^3 = \dfrac{43.3}{\pi}$, $r = 2.398$, $h = 2.398$ in

(40) $\tan\theta = \dfrac{h}{x}$ $\quad \sec^2\theta \dfrac{d\theta}{dt} = \dfrac{1}{x}\dfrac{dh}{dt}$ $\quad \dfrac{d\theta}{dt} = \dfrac{\cos^2\theta}{x}\dfrac{dh}{dt}$ $\quad \tan\theta = \dfrac{400}{100}$ $\quad \theta = 75.96°$

$\dfrac{d\theta}{dt} = \dfrac{\cos^2 75.96°}{100}(15) = 8.824 \times 10^{-3}\ \dfrac{\text{rad}}{\text{sec}}\left(\dfrac{180\ \text{deg}}{\pi\ \text{rad}}\right) = 0.506\ \dfrac{\text{deg}}{\text{sec}}$