Section 6.1 Exercises

1. A particle starts at $x = 0$ and moves along the x-axis with velocity $v(t) = 5$ for time $t \geq 0$. Where is the particle at $t = 4$?

2. A particle starts at $x = 0$ and moves along the x-axis with velocity $v(t) = 2t + 1$ for time $t \geq 0$. Where is the particle at $t = 4$?

3. A beach is eroding at a rate of 3 cubic yards per day. How much of the beach will be lost over a year (365 days)?

4. Water enters a pool at an increasing rate given by $W(t) = 1 + 3t$ gallons per minute from $t = 0$ to $t = 5$ minutes. Find the amount of water that has entered the pool over these five minutes.

5. A particle starts at $x = 0$ and moves along the x-axis with velocity $v(t) = t^2 + 1$ for time $t \geq 0$. Where is the particle at $t = 4$? Approximate the area under the curve using four rectangles of equal width and heights determined by the midpoints of the intervals, as in Example 2.

6. A particle starts at $x = 0$ and moves along the x-axis with velocity $v(t) = t^2 + 1$ for time $t \geq 0$. Where is the particle at $t = 5$? Approximate the area under the curve using five rectangles of equal width and heights determined by the midpoints of the intervals, as in Example 2.

Exercises 7–10 refer to the region R enclosed between the graph of the function $y = 2x - x^2$ and the x-axis for $0 \leq x \leq 2$.

7. **(a)** Sketch the region R.

(b) Partition $[0, 2]$ into 4 subintervals and show the four rectangles that LRAM uses to approximate the area of R. Compute the LRAM sum without a calculator.

8. Repeat Exercise 7(b) for RRAM and MRAM.

9. Using a calculator program, find the RAM sums that complete the following table.

n	LRAM_n	MRAM_n	RRAM_n
10			
50			
100			
500			

10. Make a conjecture about the area of the region R.

In Exercises 11–14, use RRAM with $n = 100$ to estimate the area of the region enclosed between the graph of f and the x-axis for $a \leq x \leq b$.

11. $f(x) = x^2 - x + 3, \quad a = 0, \quad b = 3$

12. $f(x) = \dfrac{1}{x}, \quad a = 1, \quad b = 3$

13. $f(x) = e^{-x^2}, \quad a = 0, \quad b = 2$

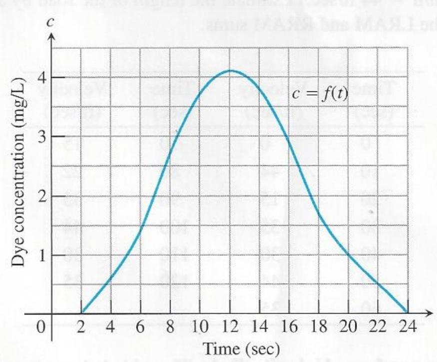
14. $f(x) = \sin x, \quad a = 0, \quad b = \pi$

15. (*Continuation of Example 4*) Use the slicing technique of Example 4 to find the MRAM sums that approximate the volume of a sphere of radius 5. Use $n = 10, 20, 40, 80$, and 160.

16. (*Continuation of Exercise 15*) Use a geometry formula to find the volume V of the sphere in Exercise 15 and find **(a)** the error and **(b)** the percentage error in the MRAM approximation for each value of n given.

17. ***Cardiac Output*** The following table gives dye concentrations for a dye-concentration cardiac-output determination like the one in Example 5. The amount of dye injected in this patient was 5 mg instead of 5.6 mg. Use rectangles to estimate the area under the dye concentration curve and then go on to estimate the patient's cardiac output.

Seconds After Injection t	Dye Concentration (adjusted for recirculation) c
2	0
4	0.6
6	1.4
8	2.7
10	3.7
12	4.1
14	3.8
16	2.9
18	1.7
20	1.0
22	0.5
24	0



18. *Distance Traveled* The table below shows the velocity of a model train engine moving along a track for 10 sec. Estimate the distance traveled by the engine, using 10 subintervals of length 1 with **(a)** left-endpoint values (LRAM) and **(b)** right-endpoint values (RRAM).

Time (sec)	Velocity (in./sec)	Time (sec)	Velocity (in./sec)
0	0	6	11
1	12	7	6
2	22	8	2
3	10	9	6
4	5	10	0
5	13		

19. *Distance Traveled Upstream* You are walking along the bank of a tidal river watching the incoming tide carry a bottle upstream. You record the velocity of the flow every 5 minutes for an hour, with the results shown in the table below. About how far upstream does the bottle travel during that hour? Find the **(a)** LRAM and **(b)** RRAM estimates using 12 subintervals of length 5.

Time (min)	Velocity (m/sec)	Time (min)	Velocity (m/sec)
0	1	35	1.2
5	1.2	40	1.0
10	1.7	45	1.8
15	2.0	50	1.5
20	1.8	55	1.2
25	1.6	60	0
30	1.4		

20. *Length of a Road* You and a companion are driving along a twisty stretch of dirt road in a car whose speedometer works but whose odometer (mileage counter) is broken. To find out how long this particular stretch of road is, you record the car's velocity at 10-sec intervals, with the results shown in the table below. (The velocity was converted from mi/h to ft/sec using 30 mi/h = 44 ft/sec.) Estimate the length of the road by averaging the LRAM and RRAM sums.

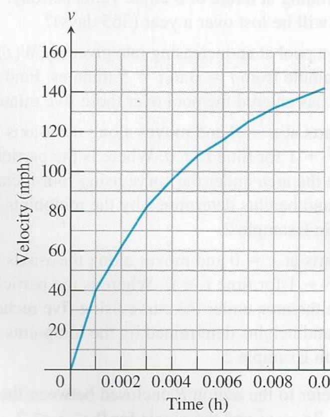
Time (sec)	Velocity (ft/sec)	Time (sec)	Velocity (ft/sec)
0	0	70	15
10	44	80	22
20	15	90	35
30	35	100	44
40	30	110	30
50	44	120	35
60	35		

21. *Distance from Velocity Data* The table below gives data for the velocity of a vintage sports car accelerating from 0 to 142 mi/h in 36 sec (10 thousandths of an hour).

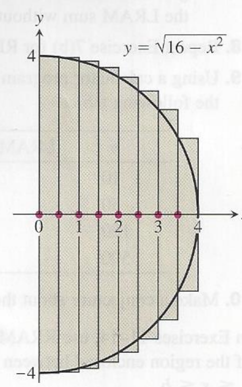
Time (h)	Velocity (mi/h)	Time (h)	Velocity (mi/h)
0.0	0	0.006	116
0.001	40	0.007	125
0.002	62	0.008	132
0.003	82	0.009	137
0.004	96	0.010	142
0.005	108		

(a) Use rectangles to estimate how far the car traveled during the 36 sec it took to reach 142 mi/h.

(b) Roughly how many seconds did it take the car to reach the halfway point? About how fast was the car going then?



22. *Volume of a Solid Hemisphere* To estimate the volume of a solid hemisphere of radius 4, imagine its axis of symmetry to be the interval $[0, 4]$ on the x-axis. Partition $[0, 4]$ into eight subintervals of equal length and approximate the solid with cylinders based on the circular cross sections of the hemisphere perpendicular to the x-axis at the subintervals' left endpoints. (See the accompanying profile view.)



(a) Writing to Learn Find the sum S_8 of the volumes of the cylinders. Do you expect S_8 to overestimate V? Give reasons for your answer.

(b) Express $|V - S_8|$ as a percentage of V to the nearest percent.

23. Repeat Exercise 22 using cylinders based on cross sections at the right endpoints of the subintervals.

24. *Volume of Water in a Reservoir* A reservoir shaped like a hemispherical bowl of radius 8 m is filled with water to a depth of 4 m.

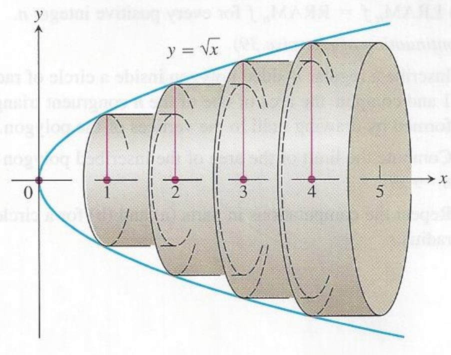
(a) Find an estimate S of the water's volume by approximating the water with eight circumscribed solid cylinders.

(b) It can be shown that the water's volume is $V = (320\pi)/3$ m^3. Find the error $|V - S|$ as a percentage of V to the nearest percent.

25. *Volume of Water in a Swimming Pool* A rectangular swimming pool is 30 ft wide and 50 ft long. The table below shows the depth $h(x)$ of the water at 5-ft intervals from one end of the pool to the other. Estimate the volume of water in the pool using **(a)** left-endpoint values and **(b)** right-endpoint values.

Position (ft) x	Depth (ft) $h(x)$	Position (ft) x	Depth (ft) $h(x)$
0	6.0	30	11.5
5	8.2	35	11.9
10	9.1	40	12.3
15	9.9	45	12.7
20	10.5	50	13.0
25	11.0		

26. *Volume of a Nose Cone* The nose "cone" of a rocket is a *paraboloid* obtained by revolving the curve $y = \sqrt{x}$, $0 \le x \le 5$ about the x-axis, where x is measured in feet. Estimate the volume V of the nose cone by partitioning $[0, 5]$ into five subintervals of equal length, slicing the cone with planes perpendicular to the x-axis at the subintervals' left endpoints, constructing cylinders of height 1 based on cross sections at these points, and finding the volumes of these cylinders. (See the accompanying figure.)



27. *Volume of a Nose Cone* Repeat Exercise 26 using cylinders based on cross sections at the *midpoints* of the subintervals.

28. *Free Fall with Air Resistance* An object is dropped straight down from a helicopter. The object falls faster and faster but its acceleration (rate of change of its velocity) decreases over time because of air resistance. The acceleration is measured in ft/sec^2 and recorded every second after the drop for 5 sec, as shown in the table below.

t	0	1	2	3	4	5
a	32.00	19.41	11.77	7.14	4.33	2.63

(a) Use LRAM_5 to find an upper estimate for the speed when $t = 5$.

(b) Use RRAM_5 to find a lower estimate for the speed when $t = 5$.

(c) Use upper estimates for the speed during the first second, second second, and third second to find an upper estimate for the distance fallen when $t = 3$.

29. *Distance Traveled by a Projectile* An object is shot straight upward from sea level with an initial velocity of 400 ft/sec.

(a) Assuming gravity is the only force acting on the object, give an upper estimate for its velocity after 5 sec have elapsed. Use $g = 32$ ft/sec^2 for the gravitational constant.

(b) Use RRAM_5 to find a lower estimate for the height attained after 5 sec.

30. *Water Pollution* Oil is leaking out of a tanker damaged at sea. The damage to the tanker is worsening, as evidenced by the increased leakage each hour, recorded in the table below.

Time (h)	0	1	2	3	4
Leakage (gal/h)	50	70	97	136	190

Time (h)	5	6	7	8
Leakage (gal/h)	265	369	516	720

(a) Give an upper and lower estimate of the total quantity of oil that has escaped after 5 hours.

(b) Repeat part (a) for the quantity of oil that has escaped after 8 hours.

(c) The tanker continues to leak 720 gal/h after the first 8 hours. If the tanker originally contained 25,000 gal of oil, approximately how many more hours will elapse in the worst case before all of the oil has leaked? in the best case?

31. *Air Pollution* A power plant generates electricity by burning oil. Pollutants produced by the burning process are removed by scrubbers in the smokestacks. Over time the scrubbers become less efficient and eventually must be replaced when the amount of pollutants released exceeds government standards. Measurements taken at the end of each month determine the rate at which pollutants are released into the atmosphere, as recorded in the table below.

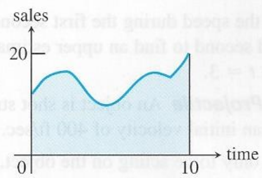
Month	Jan	Feb	Mar	Apr	May	Jun
Pollutant Release Rate (tons/day)	0.20	0.25	0.27	0.34	0.45	0.52

Month	Jul	Aug	Sep	Oct	Nov	Dec
Pollutant Release Rate (tons/day)	0.63	0.70	0.81	0.85	0.89	0.95

(a) Assuming a 30-day month and that new scrubbers allow only 0.05 ton/day released, give an upper estimate of the total tonnage of pollutants released by the end of June. What is a lower estimate?

(b) In the best case, approximately when will a total of 125 tons of pollutants have been released into the atmosphere?

32. **Writing to Learn** The graph shows the sales record for a company over a 10-year period. If sales are measured in millions of units per year, explain what information can be obtained from the area of the region, and why.



Standardized Test Questions

33. **True or False** If f is a positive, continuous, increasing function on $[a, b]$, then LRAM gives an area estimate that is less than the true area under the curve. Justify your answer.

34. **True or False** For a given number of rectangles, MRAM always gives a more accurate approximation to the true area under the curve than RRAM or LRAM. Justify your answer.

35. **Multiple Choice** If an MRAM sum with four rectangles of equal width is used to approximate the area enclosed between the x-axis and the graph of $y = 4x - x^2$, the approximation is

(A) 10 (B) 10.5 (C) $10.\overline{6}$ (D) 10.75 (E) 11

36. **Multiple Choice** If f is a positive, continuous function on an interval $[a, b]$, which of the following rectangular approximation methods has a limit equal to the actual area under the curve from a to b as the number of rectangles approaches infinity?

I. LRAM
II. RRAM
III. MRAM

(A) I and II only
(B) III only
(C) I and III only
(D) I, II, and III
(E) None of these

37. **Multiple Choice** An LRAM sum with 4 equal subdivisions is used to approximate the area under the sine curve from $x = 0$ to $x = \pi$. What is the approximation?

(A) $\frac{\pi}{4}\left(0 + \frac{\pi}{4} + \frac{\pi}{2} + \frac{3\pi}{4}\right)$ (B) $\frac{\pi}{4}\left(0 + \frac{1}{2} + \frac{\sqrt{3}}{2} + 1\right)$

(C) $\frac{\pi}{4}\left(0 + \frac{\sqrt{2}}{2} + 1 + \frac{\sqrt{2}}{2}\right)$ (D) $\frac{\pi}{4}\left(0 + \frac{1}{2} + \frac{\sqrt{2}}{2} + \frac{\sqrt{3}}{2}\right)$

(E) $\frac{\pi}{4}\left(\frac{1}{2} + \frac{\sqrt{2}}{2} + \frac{\sqrt{3}}{2} + 1\right)$

38. **Multiple Choice** A truck moves with positive velocity $v(t)$ from time $t = 3$ to time $t = 15$. The area under the graph of $y = v(t)$ between 3 and 15 gives

(A) the velocity of the truck at $t = 15$.
(B) the acceleration of the truck at $t = 15$.
(C) the position of the truck at $t = 15$.
(D) the distance traveled by the truck from $t = 3$ to $t = 15$.
(E) the average position of the truck in the interval from $t = 3$ to $t = 15$.

Exploration

39. **Group Activity** ***Area of a Circle*** Inscribe a regular n-sided polygon inside a circle of radius 1 and compute the area of the polygon for the following values of n.

(a) 4 (square) (b) 8 (octagon) (c) 16

(d) Compare the areas in parts (a), (b), and (c) with the area of the circle.

Extending the Ideas

40. ***Rectangular Approximation Methods*** Prove or disprove the following statement: MRAM_n is always the average of LRAM_n and RRAM_n.

41. ***Rectangular Approximation Methods*** Show that if f is a nonnegative function on the interval $[a, b]$ and the line $x = (a + b)/2$ is a line of symmetry of the graph of $y = f(x)$, then $\text{LRAM}_n f = \text{RRAM}_n f$ for every positive integer n.

42. *(Continuation of Exercise 39)*

(a) Inscribe a regular n-sided polygon inside a circle of radius 1 and compute the area of one of the n congruent triangles formed by drawing radii to the vertices of the polygon.

(b) Compute the limit of the area of the inscribed polygon as $n \to \infty$.

(c) Repeat the computations in parts (a) and (b) for a circle of radius r.