Section 6.2 Exercises

In Exercises 1–6, each c_k is chosen from the kth subinterval of a regular partition of the indicated interval into n subintervals of length Δx. Express the limit as a definite integral.

1. $\lim_{n\to\infty} \sum_{k=1}^{n} c_k^2 \Delta x, \quad [0, 2]$

2. $\lim_{n\to\infty} \sum_{k=1}^{n} (c_k^2 - 3c_k)\Delta x, \quad [-7, 5]$

3. $\lim_{n\to\infty} \sum_{k=1}^{n} \frac{1}{c_k} \Delta x, \quad [1, 4]$

4. $\lim_{n\to\infty} \sum_{k=1}^{n} \frac{1}{1 - c_k} \Delta x, \quad [2, 3]$

5. $\lim_{n\to\infty} \sum_{k=1}^{n} \sqrt{4 - c_k^2}\, \Delta x, \quad [0, 1]$

6. $\lim_{n\to\infty} \sum_{k=1}^{n} (\sin^3 c_k)\, \Delta x, \quad [-\pi, \pi]$

In Exercises 7–12, evaluate the integral.

7. $\int_{-2}^{1} 5\, dx$ **8.** $\int_{3}^{7} (-20)\, dx$

9. $\int_{0}^{3} (-160)\, dt$ **10.** $\int_{-4}^{-1} \frac{\pi}{2}\, d\theta$

11. $\int_{-2.1}^{3.4} 0.5\, ds$ **12.** $\int_{\sqrt{2}}^{\sqrt{18}} \sqrt{2}\, dr$

In Exercises 13–22, use the graph of the integrand and areas to evaluate the integral.

13. $\int_{-2}^{4} \left(\frac{x}{2} + 3\right) dx$ **14.** $\int_{1/2}^{3/2} (-2x + 4)\, dx$

15. $\int_{-3}^{3} \sqrt{9 - x^2}\, dx$ **16.** $\int_{-4}^{0} \sqrt{16 - x^2}\, dx$

17. $\int_{-2}^{1} |x|\, dx$ **18.** $\int_{-1}^{1} (1 - |x|)\, dx$

19. $\int_{-1}^{1} (2 - |x|)\, dx$ **20.** $\int_{-1}^{1} (1 + \sqrt{1 - x^2})\, dx$

21. $\int_{\pi}^{2\pi} \theta\, d\theta$ **22.** $\int_{\sqrt{2}}^{5\sqrt{2}} r\, dr$

In Exercises 23–28, use areas to evaluate the integral.

23. $\int_{0}^{b} x\, dx, \quad b > 0$ **24.** $\int_{0}^{b} 4x\, dx, \quad b > 0$

25. $\int_{a}^{b} 2s\, ds, \quad 0 < a < b$ **26.** $\int_{a}^{b} 3t\, dt, \quad 0 < a < b$

27. $\int_{a}^{2a} x\, dx, \quad a > 0,$ **28.** $\int_{a}^{\sqrt{3}a} x\, dx, \quad a > 0$

In Exercises 29–32, express the desired quantity as a definite integral and evaluate the integral using Theorem 2.

29. Find the distance traveled by a train moving at 87 mph from 8:00 A.M. to 11:00 A.M.

30. Find the output from a pump producing 25 gallons per minute during the first hour of its operation.

31. Find the calories burned by a walker burning 300 calories per hour between 6:00 P.M. and 7:30 P.M.

32. Find the amount of water lost from a bucket leaking 0.4 liter per hour between 8:30 A.M. and 11:00 A.M.

In Exercises 33–36, write the accumulator function $A(x)$ as a polynomial in x.

33. $A(x) = \int_{2}^{x} 3\, dt, \quad 2 \le x \le 5$

34. $A(x) = \int_{0}^{x} 3t\, dt, \quad 0 \le x \le 3$

35. $A(x) = \int_{0}^{x} (4 - 2t)\, dt, \quad 0 \le x \le 2$

36. $A(x) = \int_{1}^{x} (5 - 2t)\, dt, \quad 1 \le x \le 2$

In Exercises 37–40, use NINT to evaluate the expression.

37. $\int_{0}^{5} \frac{x}{x^2 + 4}\, dx$ **38.** $3 + 2\int_{0}^{\pi/3} \tan x\, dx$

39. Find the area enclosed between the x-axis and the graph of $y = 4 - x^2$ from $x = -2$ to $x = 2$.

40. Find the area enclosed between the x-axis and the graph of $y = x^2 e^{-x}$ from $x = -1$ to $x = 3$.

In Exercises 41–44, **(a)** find the points of discontinuity of the integrand on the interval of integration, and **(b)** use area to evaluate the integral.

41. $\int_{-2}^{3} \frac{x}{|x|}\, dx$ **42.** $\int_{-6}^{5} 2\,\mathrm{int}\,(x - 3)\, dx$

43. $\int_{-3}^{4} \frac{x^2 - 1}{x + 1}\, dx$ **44.** $\int_{-5}^{6} \frac{9 - x^2}{x - 3}\, dx$

Standardized Test Questions

45. True or False If $\int_a^b f(x)\,dx > 0$, then $f(x)$ is positive for all x in $[a, b]$. Justify your answer.

46. True or False If $f(x)$ is positive for all x in $[a, b]$, then $\int_a^b f(x)\,dx > 0$. Justify your answer.

47. Multiple Choice If $\int_2^5 f(x)\,dx = 18$, then $\int_2^5 (f(x) + 4)\,dx =$

(A) 20 (B) 22 (C) 23 (D) 25 (E) 30

48. Multiple Choice $\int_{-4}^{4} (4 - |x|)\,dx =$

(A) 0 (B) 4 (C) 8 (D) 16 (E) 32

49. Multiple Choice If the interval $[0, \pi]$ is divided into n subintervals of length π/n and c_k is chosen from the kth subinterval, which of the following is a Riemann sum?

(A) $\sum_{k=1}^{n} \sin(c_k)$ (B) $\sum_{k=1}^{\infty} \sin(c_k)$ (C) $\sum_{k=1}^{n} \sin(c_k)\left(\frac{\pi}{n}\right)$

(D) $\sum_{k=1}^{n} \sin\left(\frac{\pi}{n}\right)(c_k)$ (E) $\sum_{k=1}^{n} \sin(c_k)\left(\frac{\pi}{k}\right)$

50. Multiple Choice Which of the following quantities would *not* be represented by the definite integral $\int_0^8 70\,dt$?

(A) The distance traveled by a train moving at 70 mph for 8 minutes

(B) The volume of ice cream produced by a machine making 70 gallons per hour for 8 hours

(C) The length of a track left by a snail traveling at 70 cm per hour for 8 hours

(D) The total sales of a company selling \$70 of merchandise per hour for 8 hours

(E) The amount the tide has risen 8 minutes after low tide if it rises at a rate of 70 mm per minute during that period

Explorations

In Exercises 51–60, use graphs, your knowledge of area, and the fact that

$$\int_0^1 x^3\,dx = \frac{1}{4}$$

to evaluate the integral.

51. $\int_{-1}^{1} x^3\,dx$ **52.** $\int_0^1 (x^3 + 3)\,dx$

53. $\int_2^3 (x - 2)^3\,dx$ **54.** $\int_{-1}^{1} |x|^3\,dx$

55. $\int_0^1 (1 - x^3)\,dx$ **56.** $\int_{-1}^{2} (|x| - 1)^3\,dx$

57. $\int_0^2 \left(\frac{x}{2}\right)^3 dx$ **58.** $\int_{-8}^{8} x^3\,dx$

59. $\int_0^1 (x^3 - 1)\,dx$ **60.** $\int_0^1 \sqrt[3]{x}\,dx$

Extending the Ideas

61. Writing to Learn The function

$$f(x) = \begin{cases} \dfrac{1}{x^2}, & 0 < x \le 1 \\ 0, & x = 0 \end{cases}$$

is defined on $[0, 1]$ and has a single point of discontinuity at $x = 0$.

(a) What happens to the graph of f as x approaches 0 from the right?

(b) The function f is not integrable on $[0, 1]$. Give a convincing argument based on Riemann sums to explain why it is not.

62. It can be shown by mathematical induction (see Appendix 2) that

$$\sum_{k=1}^{n} k^2 = \frac{n(n + 1)(2n + 1)}{6}.$$

Use this fact to give a formal proof that

$$\int_0^1 x^2\,dx = \frac{1}{3}$$

by following the steps given below.

(a) Partition $[0, 1]$ into n subintervals of length $1/n$. Show that the RRAM Riemann sum for the integral is

$$\sum_{k=1}^{n} \left(\left(\frac{k}{n}\right)^2 \cdot \frac{1}{n}\right).$$

(b) Show that this sum can be written as

$$\frac{1}{n^3} \cdot \sum_{k=1}^{n} k^2.$$

(c) Show that the sum can therefore be written as

$$\frac{(n + 1)(2n + 1)}{6n^2}.$$

(d) Show that

$$\lim_{n\to\infty} \sum_{k=1}^{n} \left(\left(\frac{k}{n}\right)^2 \cdot \frac{1}{n}\right) = \frac{1}{3}.$$

(e) Explain why the equation in part (d) proves that

$$\int_0^1 x^2\,dx = \frac{1}{3}.$$