Section 6.4 Exercises

In Exercises 1–20, find dy/dx.

1. $y = \int_0^x (\sin^2 t)\, dt$

2. $y = \int_2^x (3t + \cos t^2)\, dt$

3. $y = \int_0^x (t^3 - t)^5\, dt$

4. $y = \int_{-2}^x \sqrt{1 + e^{5t}}\, dt$

5. $y = \int_2^x (\tan^3 u)\, du$

6. $y = \int_4^x e^u \sec u\, du$

7. $y = \int_7^x \frac{1 + t}{1 + t^2}\, dt$

8. $y = \int_{-\pi}^x \frac{2 - \sin t}{3 + \cos t}\, dt$

9. $y = \int_0^{x^2} e^{t^2}\, dt$

10. $y = \int_6^{x^2} \cot 3t\, dt$

11. $y = \int_2^{5x} \frac{\sqrt{1 + u^2}}{u}\, du$

12. $y = \int_\pi^{\pi - x} \frac{1 + \sin^2 u}{1 + \cos^2 u}\, du$

13. $y = \int_x^6 \ln (1 + t^2)\, dt$

14. $y = \int_x^7 \sqrt{2t^4 + t + 1}\, dt$

15. $y = \int_{x^3}^5 \frac{\cos t}{t^2 + 2}\, dt$

16. $y = \int_{5x^2}^{25} \frac{t^2 - 2t + 9}{t^3 + 6}\, dt$

17. $y = \int_{\sqrt{x}}^x \sin (r^2)\, dr$

18. $y = \int_{3x^2}^{5x} \ln (2 + p^2)\, dp$

19. $\int_{x^2}^{x^3} \cos (2t)\, dt$

20. $y = \int_{\sin x}^{\cos x} t^2\, dt$

In Exercises 21–26, construct a function of the form $y = \int_a^x f(t)\, dt + C$ that satisfies the given conditions.

21. $\frac{dy}{dx} = \sin^3 x$, and $y = 0$ when $x = 5$.

22. $\frac{dy}{dx} = e^x \tan x$, and $y = 0$ when $x = 8$.

23. $\frac{dy}{dx} = \ln (\sin x + 5)$, and $y = 3$ when $x = 2$.

24. $\frac{dy}{dx} = \sqrt{3 - \cos x}$, and $y = 4$ when $x = -3$.

25. $\dfrac{dy}{dx} = \cos^2 5x$, and $y = -2$ when $x = 7$.

26. $\dfrac{dy}{dx} = e^{\sqrt{x}}$, and $y = 1$ when $x = 0$.

In Exercises 27–40, evaluate each integral using the Evaluation Part of the Fundamental Theorem.

27. $\displaystyle\int_{1/2}^{3}\left(2 - \frac{1}{x}\right) dx$ **28.** $\displaystyle\int_{2}^{-1} 3^x\, dx$

29. $\displaystyle\int_{0}^{1} (x^2 + \sqrt{x})\, dx$ **30.** $\displaystyle\int_{0}^{5} x^{3/2}\, dx$

31. $\displaystyle\int_{1}^{32} x^{-6/5}\, dx$ **32.** $\displaystyle\int_{-2}^{-1} \frac{2}{x^2}\, dx$

33. $\displaystyle\int_{0}^{\pi} \sin x\, dx$ **34.** $\displaystyle\int_{0}^{\pi} (1 + \cos x)\, dx$

35. $\displaystyle\int_{0}^{\pi/3} 2\sec^2\theta\, d\theta$ **36.** $\displaystyle\int_{\pi/6}^{5\pi/6} \csc^2\theta\, d\theta$

37. $\displaystyle\int_{\pi/4}^{3\pi/4} \csc x \cot x\, dx$ **38.** $\displaystyle\int_{0}^{\pi/3} 4\sec x \tan x\, dx$

39. $\displaystyle\int_{-1}^{1} (r + 1)^2\, dr$ **40.** $\displaystyle\int_{1}^{4} \frac{1 - \sqrt{u}}{\sqrt{u}}\, du$

In Exercises 41–44, find the total area of the region between the curve and the x-axis.

41. $y = 2 - x, \quad 0 \le x \le 3$

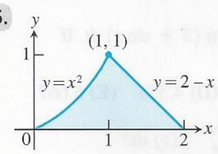
42. $y = 3x^2 - 3, \quad -2 \le x \le 2$

43. $y = x^3 - 3x^2 + 2x, \quad 0 \le x \le 2$

44. $y = x^3 - 4x, \quad -2 \le x \le 2$

In Exercises 45–48, find the area of the shaded region.

45.



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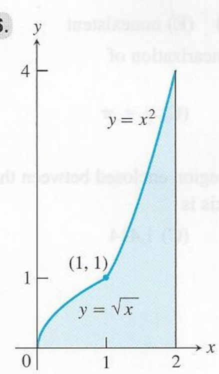

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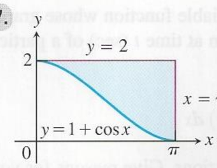

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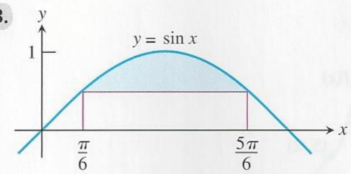


In Exercises 49–54, use NINT to solve the problem.

49. Evaluate $\displaystyle\int_{0}^{10} \frac{1}{3 + 2\sin x}\, dx.$

50. Evaluate $\displaystyle\int_{-0.8}^{0.8} \frac{2x^4 - 1}{x^4 - 1}\, dx.$

51. Find the area of the semielliptical region between the x-axis and the graph of $y = \sqrt{8 - 2x^2}$.

52. Find the average value of $\sqrt{\cos x}$ on the interval $[-1, 1]$.

53. For what value of x does $\int_0^x e^{-t^2}\, dt = 0.6$?

54. Find the area of the region in the first quadrant enclosed by the coordinate axes and the graph of $x^3 + y^3 = 1$.

In Exercises 55 and 56, find K so that

$$\int_a^x f(t)\, dt + K = \int_b^x f(t)\, dt.$$

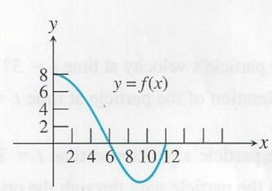
55. $f(x) = x^2 - 3x + 1; \quad a = -1; \quad b = 2$

56. $f(x) = \sin^2 x; \quad a = 0; \quad b = 2$

57. Let

$$H(x) = \int_0^x f(t)\, dt,$$

where f is the continuous function with domain $[0, 12]$ graphed here.



(a) Find $H(0)$.

(b) On what interval is H increasing? Explain.

(c) On what interval is the graph of H concave up? Explain.

(d) Is $H(12)$ positive or negative? Explain.

(e) Where does H achieve its maximum value? Explain.

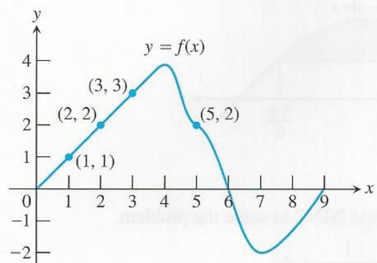
(f) Where does H achieve its minimum value? Explain.

In Exercises 58 and 59, f is the differentiable function whose graph is shown in the given figure. The position at time t (sec) of a particle moving along a coordinate axis is

$$s = \int_0^t f(x)\,dx$$

meters. Use the graph to answer the questions. Give reasons for your answers.

58.



(a) What is the particle's velocity at time $t = 5$?

(b) Is the acceleration of the particle at time $t = 5$ positive or negative?

(c) What is the particle's position at time $t = 3$?

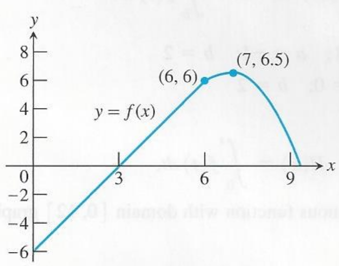
(d) At what time during the first 9 sec does s have its largest value?

(e) Approximately when is the acceleration zero?

(f) When is the particle moving toward the origin? away from the origin?

(g) On which side of the origin does the particle lie at time $t = 9$?

59.



(a) What is the particle's velocity at time $t = 3$?

(b) Is the acceleration of the particle at time $t = 3$ positive or negative?

(c) What is the particle's position at time $t = 3$?

(d) When does the particle pass through the origin?

(e) Approximately when is the acceleration zero?

(f) When is the particle moving toward the origin? away from the origin?

(g) On which side of the origin does the particle lie at time $t = 9$?

60. Suppose $\int_1^x f(t)\,dt = x^2 - 2x + 1$. Find $f(x)$.

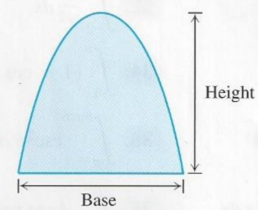
61. *Linearization* Find the linearization of

$$f(x) = 2 + \int_0^x \frac{10}{1+t}\,dt \quad \text{at} \quad x = 0.$$

62. Find $f(4)$ if $\int_0^x f(t)\,dt = x \cos \pi x$.

63. *Finding Area* Show that if k is a positive constant, then the area between the x-axis and one arch of the curve $y = \sin kx$ is always $2/k$.

64. *Archimedes' Area Formula for Parabolas* Archimedes (287–212 B.C.), inventor, military engineer, physicist, and the greatest mathematician of classical times, discovered that the area under a parabolic arch like the one shown here is always two-thirds the base times the height.



(a) Find the area under the parabolic arch

$$y = 6 - x - x^2, \quad -3 \le x \le 2.$$

(b) Find the height of the arch.

(c) Show that the area is two-thirds the base times the height.

Standardized Test Questions

You may use a graphing calculator to solve the following problems.

65. True or False If f is continuous on an open interval I containing a, then F defined by $F(x) = \int_a^x f(t)\,dt$ is continuous on I. Justify your answer.

66. True or False If $b > a$, then $\dfrac{d}{dx}\int_a^b e^{x^2}\,dx$ is positive. Justify your answer.

67. Multiple Choice Let $f(x) = \int_a^x \ln(2 + \sin t)\,dt$. If $f(3) = 4$, then $f(5) =$

(A) 0.040 **(B)** 0.272 **(C)** 0.961 **(D)** 4.555 **(E)** 6.667

68. Multiple Choice What is $\lim_{h \to 0} \frac{1}{h} \int_x^{x+h} f(t)\,dt$?

(A) 0 **(B)** 1 **(C)** $f'(x)$ **(D)** $f(x)$ **(E)** nonexistent

69. Multiple Choice At $x = \pi$, the linearization of $f(x) = \int_\pi^x \cos^3 t\,dt$ is

(A) $y = -1$ **(B)** $y = -x$ **(C)** $y = \pi$

(D) $y = x - \pi$ **(E)** $y = \pi - x$

70. Multiple Choice The area of the region enclosed between the graph of $y = \sqrt{1 - x^4}$ and the x-axis is

(A) 0.886 **(B)** 1.253 **(C)** 1.414

(D) 1.571 **(E)** 1.748

Explorations

71. *The Sine Integral Function* The sine integral function

$$\text{Si}(x) = \int_0^x \frac{\sin t}{t}\,dt$$

is one of the many useful functions in engineering that are defined as integrals. Although the notation does not show it, the function being integrated is

$$f(t) = \begin{cases} \dfrac{\sin t}{t}, & t \neq 0 \\ 1, & t = 0, \end{cases}$$

the continuous extension of $(\sin t)/t$ to the origin.

(a) Show that $\text{Si}(x)$ is an odd function of x.

(b) What is the value of $\text{Si}(0)$?

(c) Find the values of x at which $\text{Si}(x)$ has a local extreme value.

(d) Use NINT to graph $\text{Si}(x)$.

72. *Cost from Marginal Cost* The marginal cost of printing a poster when x posters have been printed is

$$\frac{dc}{dx} = \frac{1}{2\sqrt{x}}$$

dollars. Find

(a) $c(100) - c(1)$, the cost of printing posters 2 to 100.

(b) $c(400) - c(100)$, the cost of printing posters 101 to 400.

73. *Revenue from Marginal Revenue* Suppose that a company's marginal revenue from the manufacture and sale of eggbeaters is

$$\frac{dr}{dx} = 2 - \frac{2}{(x+1)^2},$$

where r is measured in thousands of dollars and x in thousands of units. How much money should the company expect from a production run of $x = 3$ thousand eggbeaters? To find out, integrate the marginal revenue from $x = 0$ to $x = 3$.

74. *Average Daily Holding Cost* Solon Container receives 450 drums of plastic pellets every 30 days. The inventory function (drums on hand as a function of days) is $I(x) = 450 - x^2/2$.

(a) Find the average daily inventory (that is, the average value of $I(x)$ for the 30-day period).

(b) If the holding cost for one drum is \$0.02 per day, find the average daily holding cost (that is, the per-drum holding cost times the average daily inventory).

75. Suppose that f has a negative derivative for all values of x and that $f(1) = 0$. Which of the following statements must be true of the function

$$h(x) = \int_0^x f(t)\,dt?$$

Give reasons for your answers.

(a) h is a twice-differentiable function of x.

(b) h and dh/dx are both continuous.

(c) The graph of h has a horizontal tangent at $x = 1$.

(d) h has a local maximum at $x = 1$.

(e) h has a local minimum at $x = 1$.

(f) The graph of h has an inflection point at $x = 1$.

(g) The graph of dh/dx crosses the x-axis at $x = 1$.

Extending the Ideas

76. Writing to Learn If f is an odd continuous function, give a graphical argument to explain why $\int_0^x f(t)\,dt$ is even.

77. Writing to Learn If f is an even continuous function, give a graphical argument to explain why $\int_0^x f(t)\,dt$ is odd.

78. Writing to Learn Explain why we can conclude from Exercises 76 and 77 that every even continuous function is the derivative of an odd continuous function and vice versa.

79. Give a convincing argument that the equation

$$\int_0^x \frac{\sin t}{t}\,dt = 1$$

has exactly one solution. Give its approximate value.