Section 6.5 Exercises

In Exercises 1–6, **(a)** use the Trapezoidal Rule with $n = 4$ to approximate the value of the integral. **(b)** Use the concavity of the function to predict whether the approximation is an overestimate or an underestimate. Finally, **(c)** find the integral's exact value to check your answer.

1. $\displaystyle\int_0^2 x\,dx$ **2.** $\displaystyle\int_0^2 x^2\,dx$

3. $\displaystyle\int_0^2 x^3\,dx$ **4.** $\displaystyle\int_1^2 \frac{1}{x}\,dx$

5. $\displaystyle\int_0^4 \sqrt{x}\,dx$ **6.** $\displaystyle\int_0^{\pi} \sin x\,dx$

7. Use the function values in the following table and the Trapezoidal Rule with $n = 6$ to approximate $\int_0^6 f(x)\,dx$.

x	0	1	2	3	4	5	6
$f(x)$	12	10	9	11	13	16	18

8. Use the function values in the following table and the Trapezoidal Rule with $n = 6$ to approximate $\int_2^8 f(x)\,dx$.

x	2	3	4	5	6	7	8
$f(x)$	16	19	17	14	13	16	20

9. ***Volume of Water in a Swimming Pool*** A rectangular swimming pool is 30 ft wide and 50 ft long. The table below shows the depth $h(x)$ of the water at 5-ft intervals from one end of the pool to the other. Estimate the volume of water in the pool using the Trapezoidal Rule with $n = 10$, applied to the integral

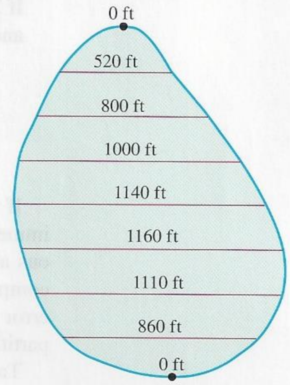
$$V = \int_0^{50} 30 \cdot h(x)\,dx.$$

Position (ft) x	Depth (ft) $h(x)$	Position (ft) x	Depth (ft) $h(x)$
0	6.0	30	11.5
5	8.2	35	11.9
10	9.1	40	12.3
15	9.9	45	12.7
20	10.5	50	13.0
25	11.0		

10. ***Stocking a Fish Pond*** As the fish and game warden of your township, you are responsible for stocking the town pond with fish before the fishing season. The average depth of the pond is 20 feet. Using a scaled map, you measure distances across the pond at 200-foot intervals, as shown in the diagram.

(a) Use the Trapezoidal Rule to estimate the volume of the pond.

(b) You plan to start the season with one fish per 1000 cubic feet. You intend to have at least 25% of the opening day's fish population left at the end of the season. What is the maximum number of licenses the town can sell if the average seasonal catch is 20 fish per license?



Vertical spacing = 200 ft

11. ***Scarpellone Panther I*** The accompanying table shows time-to-speed data for a 2011 Scarpellone Panther I accelerating from rest to 130 mph. How far had the Panther I traveled by the time it reached this speed? (Use trapezoids to estimate the area under the velocity curve, but be careful: the time intervals vary in length.)

Speed Change: Zero to	Time (sec)
30 mph	1.8
40 mph	3.1
50 mph	4.2
60 mph	5.5
70 mph	7.4
80 mph	9.2
90 mph	11.5
100 mph	14.6
120 mph	20.9
130 mph	25.7

12. The table below records the velocity of a bobsled at 1-second intervals for the first eight seconds of its run. Use the Trapezoidal Rule to approximate the distance the bobsled travels during that 8-second interval. (Give your final answer in feet.)

Time (seconds)	Speed (miles/hr)
0	0
1	3
2	7
3	12
4	17
5	25
6	33
7	41
8	48

In Exercises 13–18, **(a)** use Simpson's Rule with $n = 4$ to approximate the value of the integral and **(b)** find the exact value of the integral to check your answer. (Note that these are the same integrals as Exercises 1–6, so you can also compare it with the Trapezoidal Rule approximation.)

13. $\int_0^2 x\,dx$ **14.** $\int_0^2 x^2\,dx$

15. $\int_0^2 x^3\,dx$ **16.** $\int_1^2 \frac{1}{x}\,dx$

17. $\int_0^4 \sqrt{x}\,dx$ **18.** $\int_0^{\pi} \sin x\,dx$

19. Consider the integral $\int_{-1}^{3}(x^3 - 2x)\,dx$.

(a) Use Simpson's Rule with $n = 4$ to approximate its value.

(b) Find the exact value of the integral. What is the error, $|E_S|$?

(c) Explain how you could have predicted what you found in (b) from knowing the error-bound formula.

(d) **Writing to Learn** Is it possible to make a general statement about using Simpson's Rule to approximate integrals of cubic polynomials? Explain.

20. **Writing to Learn** In Example 2 (before rounding) we found the average temperature to be 65.17 degrees when we used the integral approximation, yet the average of the 13 discrete temperatures is only 64.69 degrees. Considering the shape of the temperature curve, explain why you would expect the average of the 13 discrete temperatures to be less than the average value of the temperature function on the entire interval.

21. *(Continuation of Exercise 20)*

(a) In the Trapezoidal Rule, every function value in the sum is doubled except for the two endpoint values. Show that if you double the endpoint values, you get 70.08 for the average temperature.

(b) Explain why it makes more sense to not double the endpoint values if we are interested in the average temperature over the entire 12-hour period.

22. **Group Activity** For most functions, Simpson's Rule gives a better approximation to an integral than the Trapezoidal Rule for a given value of n. Sketch the graph of a function on a closed interval for which the Trapezoidal Rule obviously gives a better approximation than Simpson's Rule for $n = 4$.

In Exercises 23–26, use a calculator program to find the Simpson's Rule approximations with $n = 50$ and $n = 100$.

23. $\int_{-1}^{1} 2\sqrt{1 - x^2}\,dx$

24. $\int_0^1 \sqrt{1 + x^4}\,dx$

25. $\int_0^{\pi/2} \frac{\sin x}{x}\,dx$

26. $\int_0^{\pi/2} \sin(x^2)\,dx$

27. Consider the integral $\int_0^{\pi} \sin x\,dx$.

(a) Use a calculator program to find the Trapezoidal Rule approximations for $n = 10$, 100, and 1000.

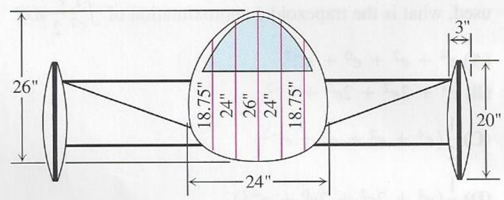
(b) Record the errors with as many decimal places of accuracy as you can.

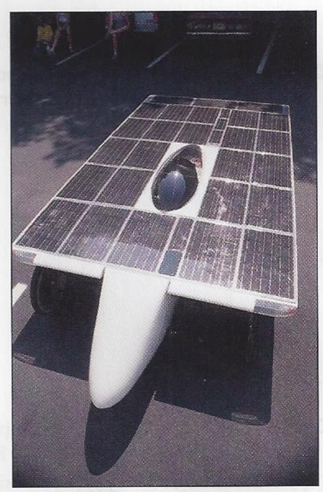
(c) What pattern do you see?

(d) **Writing to Learn** Explain how the error bound for E_T accounts for the pattern.

28. *(Continuation of Exercise 27)* Repeat Exercise 27 with Simpson's Rule and E_S.

29. ***Aerodynamic Drag*** A vehicle's aerodynamic drag is determined in part by its cross-section area, so, all other things being equal, engineers try to make this area as small as possible. Use Simpson's Rule to estimate the cross-section area of the body of James Worden's solar-powered Solectria® automobile at M.I.T. from the diagram below.





30. ***Wing Design*** The design of a new airplane requires a gasoline tank of constant cross-section area in each wing. A scale drawing of a cross section is shown here. The tank must hold 5000 lb of gasoline, which has a density of 42 lb/ft^3. Estimate the length of the tank.

$y_0 = 1.5$ ft, $y_1 = 1.6$ ft, $y_2 = 1.8$ ft, $y_3 = 1.9$ ft,
$y_4 = 2.0$ ft, $y_5 = y_6 = 2.1$ ft Horizontal spacing = 1 ft

Standardized Test Questions

31. True or False The Trapezoidal Rule will underestimate $\int_a^b f(x)\,dx$ if the graph of f is concave up on $[a, b]$. Justify your answer.

32. True or False For a given value of n, the Trapezoidal Rule with n subdivisions will always give a more accurate estimate of $\int_a^b f(x)\,dx$ than a right Riemann sum with n subdivisions. Justify your answer.

33. Multiple Choice Using 8 equal subdivisions of the interval $[2, 12]$, the LRAM approximation of $\int_2^{12} f(x)\,dx$ is 16.6 and the trapezoidal approximation is 16.4. What is the RRAM approximation?

(A) 16.2 (B) 16.5
(C) 16.6 (D) 16.8
(E) It cannot be determined from the given information.

34. Multiple Choice If three equal subdivisions of $[-2, 4]$ are used, what is the trapezoidal approximation of $\int_{-2}^{4} \frac{e^x}{2}\,dx$?

(A) $e^4 + e^2 + e^0 + e^{-2}$
(B) $e^4 + 2e^2 + 2e^0 + e^{-2}$
(C) $\frac{1}{2}(e^4 + e^2 + e^0 + e^{-2})$
(D) $\frac{1}{2}(e^4 + 2e^2 + 2e^0 + e^{-2})$
(E) $\frac{1}{4}(e^4 + 2e^2 + 2e^0 + e^{-2})$

35. Multiple Choice The trapezoidal approximation of $\int_0^{\pi} \sin x\,dx$ using 4 equal subdivisions of the interval of integration is

(A) $\frac{\pi}{2}$
(B) π
(C) $\frac{\pi}{4}(1 + \sqrt{2})$
(D) $\frac{\pi}{2}(1 + \sqrt{2})$
(E) $\frac{\pi}{4}(2 + \sqrt{2})$

36. Multiple Choice Suppose f, f', and f'' are all positive on the interval $[a, b]$, and suppose we compute LRAM, RRAM, and trapezoidal approximations of $I = \int_a^b f(x)\,dx$ using the same number of equal subdivisions of $[a, b]$. If we denote the three approximations of I as L, R, and T respectively, which of the following is true?

(A) $R < T < I < L$ (B) $R < I < T < L$
(C) $L < I < T < R$ (D) $L < T < I < R$
(E) $L < I < R < T$

Explorations

37. Consider the integral $\int_{-1}^{1} \sin(x^2)\,dx$.

(a) Find f'' for $f(x) = \sin(x^2)$.

(b) Graph $y = f''(x)$ in the viewing window $[-1, 1]$ by $[-3, 3]$.

(c) Explain why the graph in part (b) suggests that $|f''(x)| \le 3$ for $-1 \le x \le 1$.

(d) Show that the error estimate for the Trapezoidal Rule in this case becomes

$$|E_T| \le \frac{h^2}{2}.$$

(e) Show that the Trapezoidal Rule error will be less than or equal to 0.01 if $h \le 0.1$.

(f) How large must n be for $h \le 0.1$?

38. Consider the integral $\int_{-1}^{1} \sin(x^2)\,dx$.

(a) Find $f^{(4)}$ for $f(x) = \sin(x^2)$. (You may want to check your work with a CAS if you have one available.)

(b) Graph $y = f^{(4)}(x)$ in the viewing window $[-1, 1]$ by $[-30, 10]$.

(c) Explain why the graph in part (b) suggests that $|f^{(4)}(x)| \le 30$ for $-1 \le x \le 1$.

(d) Show that the error estimate for Simpson's Rule in this case becomes

$$|E_S| \le \frac{h^4}{3}.$$

(e) Show that the Simpson's Rule error will be less than or equal to 0.01 if $h \le 0.4$.

(f) How large must n be for $h \le 0.4$?

Extending the Ideas

39. Using the definitions, prove that, in general,

$$T_n = \frac{\text{LRAM}_n + \text{RRAM}_n}{2}.$$

40. Using the definitions, prove that, in general,

$$S_{2n} = \frac{\text{MRAM}_n + 2T_{2n}}{3}.$$