AP CALCULUS AB

INTEGRALS OF PIECEWISE-DEFINED FUNCTIONS

For problems **1** and **2**, given $y = f'(x)$:

a) Graph $y = f'(x)$ on the grid provided.

b) Calculate $y = f(x)$ for $x \in [0\,, 8]$ using the initial condition $f(0) = 0$.

c) Graph $y = f(x)$ on the grid provided.

d) Is $f(x)$ differentiable at $x = 4$?

1)

$$f'(x) = \begin{cases} \dfrac{1}{2}x + 1 \quad , & 0 \le x < 4 \\ \dfrac{1}{2}x - 1 \quad , & 4 < x \le 8 \end{cases}$$

$0 \le x \le 4$:

$$f(x) = \cancel{f(0)} + \int_0^x f'(t)\,dt =$$

$$= \int_0^x \left(\tfrac{1}{2}t + 1\right) dt = \left[\tfrac{1}{4}t^2 + t\right]_0^x =$$

$$= \tfrac{1}{4}x^2 + x. \quad f(4) = 8$$

$4 < x \le 8$:

$$f(x) = f(4) + \int_4^x f'(t)\,dt =$$

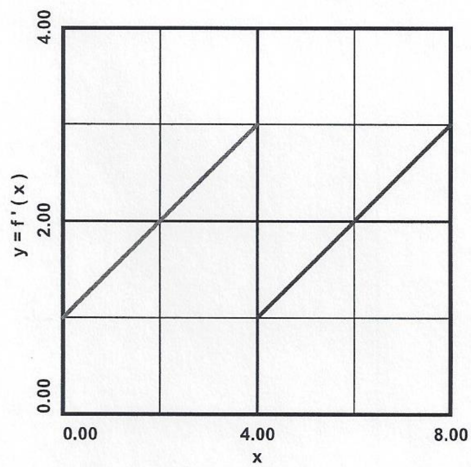
$$= 8 + \int_4^x \left(\tfrac{1}{2}t - 1\right) dt =$$

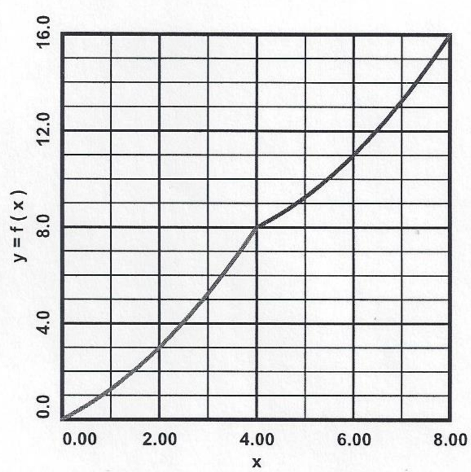
$$= 8 + \left[\tfrac{1}{4}t^2 - t\right]_4^x =$$

$$= 8 + \tfrac{1}{4}x^2 - x - 0 =$$

$$= \tfrac{1}{4}x^2 - x + 8$$

$$f(x) = \begin{cases} \tfrac{1}{4}x^2 + x \;, & 0 \le x \le 4 \\ \tfrac{1}{4}x^2 - x + 8 \;, & 4 < x \le 8 \end{cases}$$





$y = f(x)$ is not differentiable at $x = 4$ due to the kink at $x = 4$

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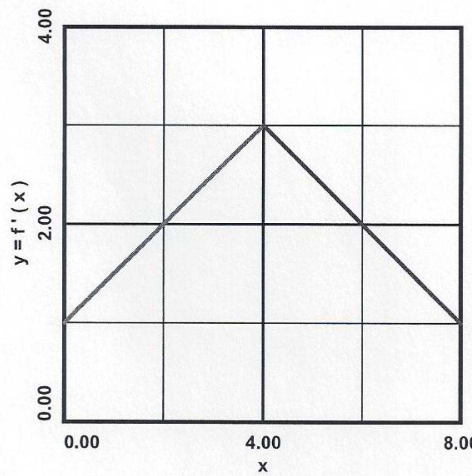
2)

$$f'(x) = \begin{cases} \frac{1}{2}x + 1 & , \quad 0 \le x \le 4 \\ -\frac{1}{2}x + 5 & , \quad 4 < x \le 8 \end{cases}$$



$0 \le x \le 4$:

$f(x) = \frac{1}{4}x^2 + x \qquad f(4) = 8$

$4 < x \le 8$:

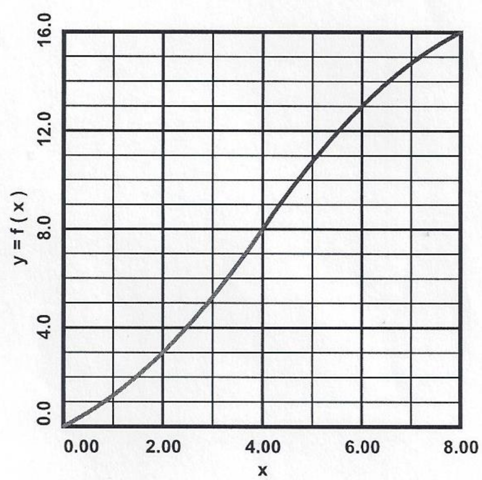
$$f(x) = f(4) + \int_4^x f'(t)\,dt =$$

$$= 8 + \int_4^x \left(-\frac{1}{2}t + 5\right)dt =$$

$$= 8 + \left[-\frac{1}{4}t^2 + 5t\right]_4^x =$$

$$= 8 - \frac{1}{4}x^2 + 5x - 16 =$$

$$= -\frac{1}{4}x^2 + 5x - 8$$



$$f(x) = \begin{cases} \frac{1}{4}x^2 + x & , \quad 0 \le x \le 4 \\ -\frac{1}{4}x^2 + 5x - 8 & , \quad 4 < x \le 8 \end{cases}$$

$y = f(x)$ is differentiable at $x = 4$. In fact $f(4^-) = 3$ and $f(4^+) = 3$