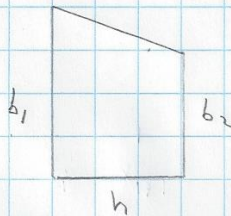


6.5. Trapezoidal Rule

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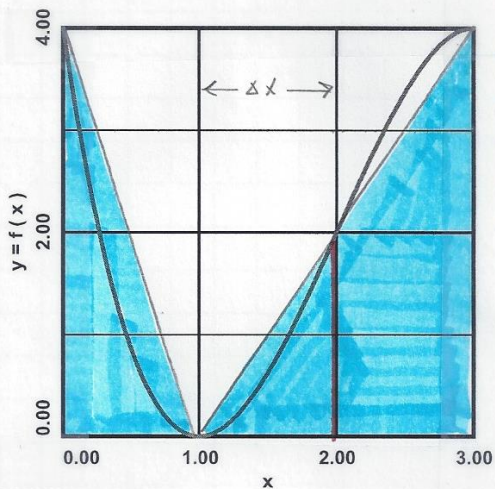
Area of Trapezoid

$$\text{area} = \frac{1}{2} (b_1 + b_2) h$$



Example: Estimate the area under $f(x) = -x^3 + 6x^2 - 9x + 4$ on $x \in [0, 3]$ using 3 equal subdivisions with 3 trapezoids. Draw the trapezoids on the graph of the function. Also, what is the exact value of the area?

SOLUTION:



$$\Delta x = 1 \equiv \text{subdivision size}$$

$$\text{area} \approx$$

$$\begin{aligned} & \frac{1}{2} [f(0) + f(1)] \Delta x \\ & + \frac{1}{2} [f(1) + f(2)] \Delta x \\ & + \frac{1}{2} [f(2) + f(3)] \Delta x = \\ & = \frac{1}{2} [f(0) + 2f(1) + 2f(2) + f(3)] \Delta x \\ & = 6 \end{aligned}$$

$$\text{The exact value is } \text{area} = \int_0^3 (-x^3 + 6x^2 - 9x + 4) dx = \left[-\frac{1}{4}x^4 + 2x^3 - \frac{9}{2}x^2 + 4x \right]_0^3 = 5.25$$