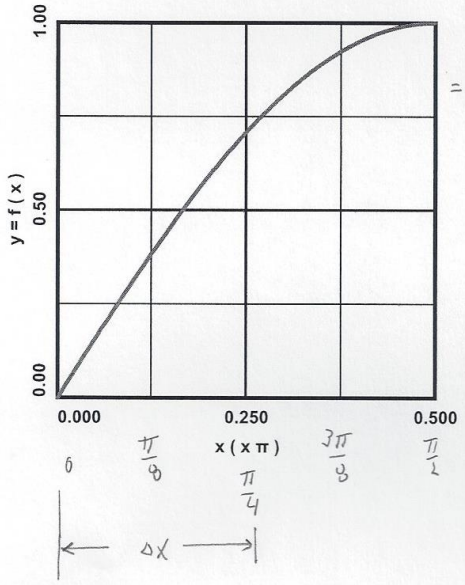
AP CALCULUS AB

SIMPSON'S RULE

Estimate the value of

$$\int_0^{\pi/2} \sin x \, dx$$

using Simpson's Rule with two equal subdivisions. Also, calculate the percent relative error of the estimation.



$\Delta x = \frac{\pi}{4} \equiv$ subdivision size $\quad f(x) = \sin x$

$$\text{area} \approx \frac{\Delta x}{6}\left[f(0) + 4f\left(\frac{\pi}{8}\right) + f\left(\frac{\pi}{4}\right)\right]$$

$$+ \frac{\Delta x}{6}\left[f\left(\frac{\pi}{4}\right) + 4f\left(\frac{3\pi}{8}\right) + f\left(\frac{\pi}{2}\right)\right] =$$

$$= \frac{\Delta x}{6}\left[f(0) + 4f\left(\frac{\pi}{8}\right) + 2f\left(\frac{\pi}{4}\right) + 4f\left(\frac{3\pi}{8}\right) + f\left(\frac{\pi}{2}\right)\right] =$$

$$= 1.000\ 134\ 585$$

$$\int_0^{\pi/2} \sin x \, dx = \left[-\cos x\right]_0^{\frac{\pi}{2}} = -\cos\frac{\pi}{2} + \cos 0 = 0 + 1 = 1$$

$$\text{error} = \frac{1.000\ 134\ 855 - 1}{1} = 0.0135\%$$