6.5. Simpson's Rule

We derived that

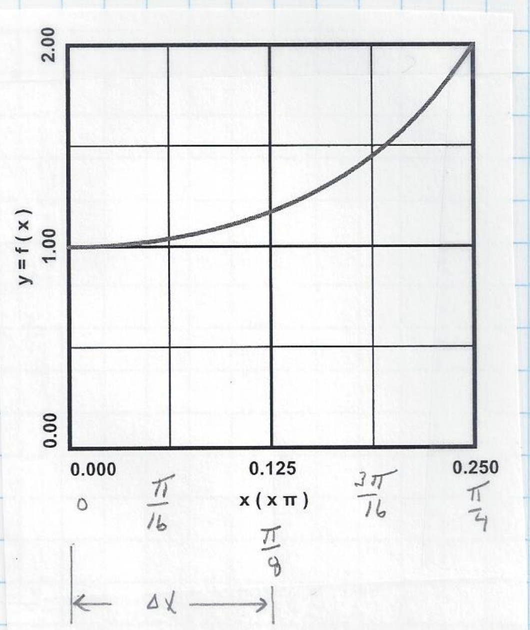
$$\int_A^B f(x)\,dx \approx \sum_{i=1}^{N} \frac{1}{6}\left[f(x_i) + 4f(x_{i+0.5}) + f(x_{i+1}) \right] \Delta x$$

where

$$\Delta x = \frac{B-A}{N}, \quad x_i = A + (i-1)\Delta x, \quad x_{i+0.5} = x_i + \frac{\Delta x}{2}, \quad x_{i+1} = x_i + \Delta x.$$

Example. Estimate $\int_0^{\frac{\pi}{4}} \sec^2 x\,dx$ with Simpson's Rule using 2 equal subdivisions. Also, calculate the percent relative area of the estimation.

SOLUTION: $f(x) = \sec^2 x$



$\Delta x = \frac{\pi}{8} \equiv$ subdivision size

$$\text{area} \approx \frac{\Delta x}{6}\left[f(0) + 4f\left(\frac{\pi}{16}\right) + f\left(\frac{\pi}{8}\right) \right] + \frac{\Delta x}{6}\left[f\left(\frac{\pi}{8}\right) + 4f\left(\frac{3\pi}{16}\right) + f\left(\frac{\pi}{4}\right) \right]$$

$$= \frac{\Delta x}{6}\left[f(0) + 4f\left(\frac{\pi}{16}\right) + 2f\left(\frac{\pi}{8}\right) + 4f\left(\frac{3\pi}{16}\right) + f\left(\frac{\pi}{4}\right) \right]$$

$$= 1.000\,548\,895$$

$$\int_0^{\pi/4} \sec^2 x\,dx = \left[\tan x\right]_0^{\pi/4} = \tan\frac{\pi}{4} - \tan 0 =$$

$$= 1 - 0 = 1$$

$$\text{error} = \frac{1.000\,548\,895 - 1}{1} = 0.0549\,\%$$