AP CALCULUS AB

HOMEWORK #10

6.3. Average Value of a Function

Section 6.3 Exercises, pg. 298
32, 33, 35

Supplemental Problems
1

6.4. Integrals of Piecewise-Defined Functions

Supplemental Problems
2, **3**

6.5. Trapezoidal Rule

Section 6.5 Exercises, pg. 320
2, 7, 10

6.5. Trapezoidal Rule Program

Section 6.5 Exercises, pg. 320
27(a,b)[1]

6.5. Derivation of Simpson's Rule

Supplemental Problems
4

6.5. Simpson's Rule

Section 6.5 Exercises, pg. 320
16[2], 29

6.5. Program for Simpson's Rule

Supplemental Problems
5

Notes:

1. Add a part (c), *viz.*, to two significant digits, what do you think the absolute error will be for 10,000 subdivisions?

2. Use two equal subdivisions of size $\Delta x = 0.5$.

Supplemental Problems:

1) For $f(x) = x^3 - 6x^2 + 9x$ defined on $x \in [0\,,4]$, find the values of $c \in [0\,,4]$ which are guaranteed to exist via the Mean Value Theorem for Definite Integrals. *Hint:* one of the values is $c = 2$. Find the other two values by using synthetic division and the Quadratic Formula.

For problems **2** and **3**, given $y = f'(x)$,

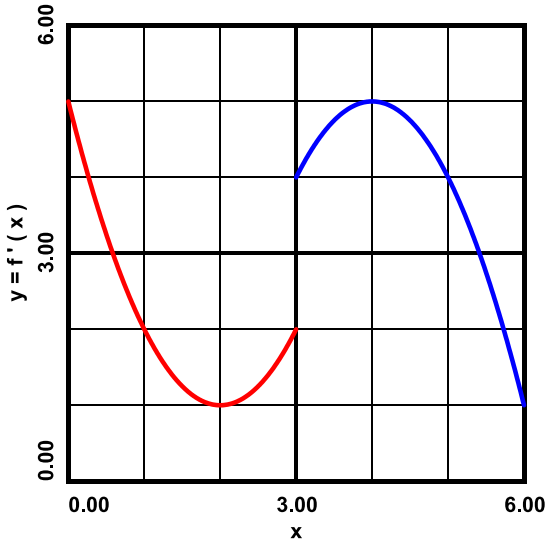
a) State whether or not $y = f(x)$ is differentiable at $x = 3$, and whether $y = f(x)$ has a kink there or is smooth there.

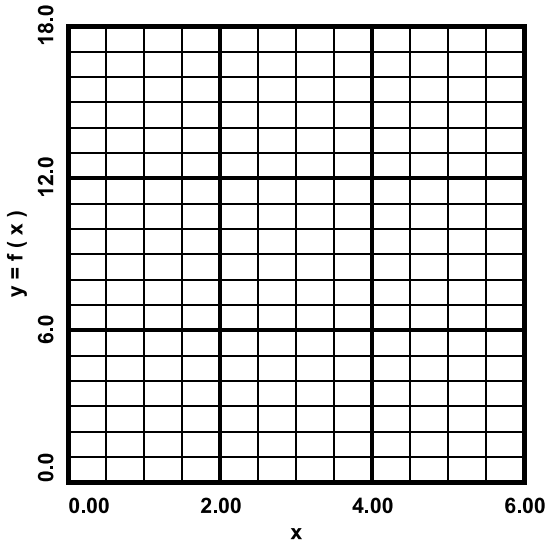
b) Calculate $y = f(x)$ subject to the initial condition $f(0) = 0$.

c) Graph $y = f(x)$ on the grid provided.

2)

$$f'(x) = \begin{cases} x^2 - 4x + 5 & , \quad 0 \le x < 3 \\ -x^2 + 8x - 11 & , \quad 3 < x \le 6 \end{cases}$$



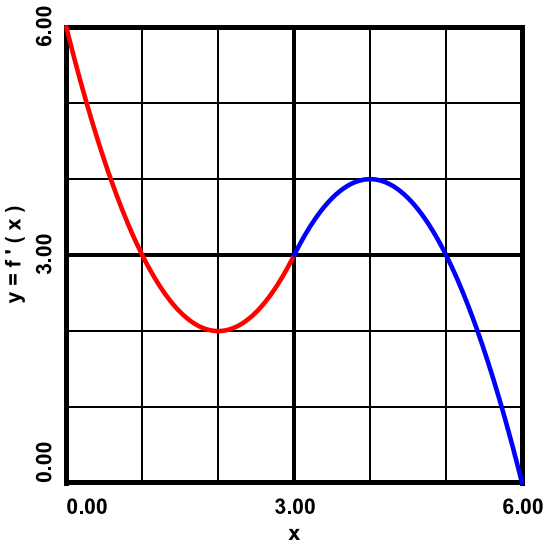


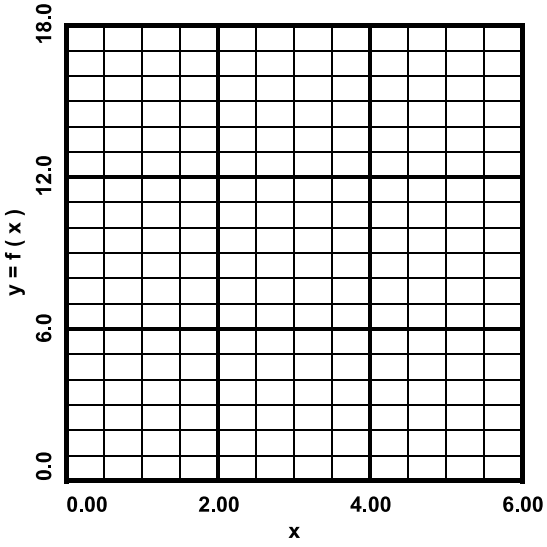
AP CALCULUS AB

HOMEWORK #10

3)

$$f'(x) = \begin{cases} x^2 - 4x + 6 & , \quad 0 \le x \le 3 \\ -x^2 + 8x - 12 & , \quad 3 < x \le 6 \end{cases}$$

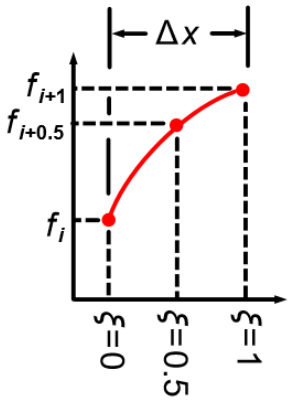




4) When we derived Simpson's Rule in lecture, we started by verifying that

$$f(\xi) = (2f_i - 4f_{i+0.5} + 2f_{i+1})\xi^2 + \\ +(-3f_i + 4f_{i+0.5} - f_{i+1})\xi + f_i$$

corresponds to the figure below.



Derive this quadratic by assuming a function of the form $f(\xi) = a\xi^2 + b\xi + c$, and then by using the conditions $f(0) = f_i$, $f(0.5) = f_{i+0.5}$ and $f(1) = f_{i+1}$ to solve for the coefficients a, b and c.

5) For the integral

$$I = \int_0^{\pi/3} \sqrt{3}\,\sec^2 x \; dx\,,$$

a) Calculate the exact value of I.

b) Estimate I using `prgmSR` with 2, 10, 100 and 200 subdivisions.

c) Evaluate I using `fnInt`.