HW #10

pg. 298

6.3. Average Value of a Function

(32) $f(x)=\frac{1}{x}$, $\bar{f}=\frac{1}{e}\int_e^{2e}\frac{dx}{x}=\frac{1}{e}\left[\ln x\right]_e^{2e}=\frac{1}{e}\left[\ln(2e)-\ln(e)\right]=\frac{1}{e}\ln\left(\frac{2e}{e}\right)=$

$=\frac{1}{e}\ln 2\approx 0.255$

(33) $f(x)=\sec^2 x$, $\bar{f}=\frac{1}{\pi/4}\int_0^{\pi/4}\sec^2 x\,dx=\frac{4}{\pi}\left[\tan x\right]_0^{\pi/4}=\frac{4}{\pi}\left[\tan\left(\frac{\pi}{4}\right)-\tan 0\right]=$

$=\frac{4}{\pi}\approx 1.273$

(35) $f(x)=3x^2+2x$, $\bar{f}=\frac{1}{3}\int_{-1}^{2}(3x^2+2x)\,dx=\frac{1}{3}\left[x^3+x^2\right]_{-1}^{2}=\frac{1}{3}\left[12-0\right]=4$

Supplemental

(1) $f(x)=x^3-6x^2+9x$ on $x\in[0,4]$, $\bar{f}=\frac{1}{4}\int_0^4(x^3-6x^2+9x)\,dx=$

$=\frac{1}{4}\left[\frac{1}{4}x^4-2x^3+\frac{9}{2}x^2\right]_0^4=\frac{1}{4}\cdot 8=2$, $c^3-6c^2+9c=2$,

$c^3-6c^2+9c-2=0$. $c=2$ is one root

$$\begin{array}{r|rrr|r} 2 & 1 & -6 & 9 & -2 \\ & & 2 & -8 & 2 \\ \hline & 1 & -4 & 1 & 0 \end{array}$$

$c^2-4c+1=0$

$c=\frac{4\pm\sqrt{4^2-4(1)(1)}}{2(1)}=\frac{4\pm\sqrt{12}}{2}=\frac{4\pm2\sqrt{3}}{2}=2\pm\sqrt{3}$ $c=0.268$, $c=3.732$

Supplemental

6.4. Integral of Piecewise-Defined Function

(2) $f'(x)=\begin{cases}x^2-4x+5, & 0\le x<3\\ -x^2+8x-11, & 3<x\le 6\end{cases}$ (a) $\left.\begin{array}{l}f'(3^-)=2\\ f'(3^+)=4\end{array}\right\}$ $\Rightarrow$ so $f(x)$ is not differentiable at $x=3$, i.e., $f(x)$ will have a kink at $x=3$.

(b) $0\le x<3$:

$f(x)=\cancel{f(0)}+\int_0^x f'(t)\,dt=\int_0^x(t^2-4t+5)\,dt=\left[\frac{1}{3}t^3-2t^2+5t\right]_0^x=\frac{1}{3}x^3-2x^2+5x$

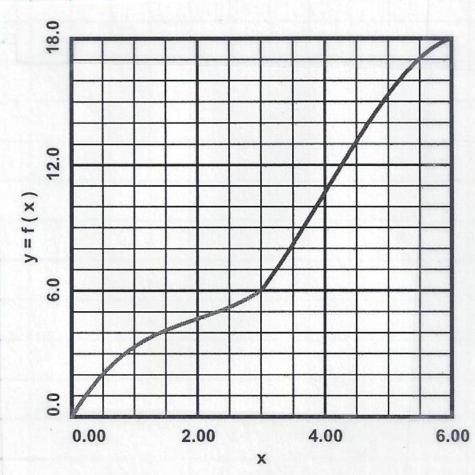
$f(3)=6$. $3<x\le 6$: $f(x)=f(3)+\int_3^x f'(t)\,dt=6+\int_3^x(-t^2+8t-11)\,dt=$

$=6+\left[-\frac{1}{3}t^3+4t^2-11t\right]_3^x=6+\left[-\frac{1}{3}x^3+4x^2-11x--6\right]=-\frac{1}{3}x^3+4x^2-11x+12$

$f(x)=\begin{cases}\frac{1}{3}x^3-2x^2+5x, & 0\le x\le 3\\ -\frac{1}{3}x^3+4x^2-11x+12, & 3<x\le 6\end{cases}$

HW #10

(c)



(3) $f'(x) = \begin{cases} x^2 - 4x + 6 , & 0 \le x \le 3 \\ -x^2 + 8x - 12 , & 3 < x \le 6 \end{cases}$

(a) $\left.\begin{array}{l} f'(3^-) = 3 \\ f'(3^+) = 3 \end{array}\right\} \Rightarrow$ f(x) is differentiable at $x = 3$ ◄ i.e., f(x) will be smooth at $x = 3$.

(b) $0 \le x \le 3$:

$f(x) = \cancel{f(0)}^{\,0} + \int_0^x f'(t)\,dt =$

$= \int_0^x (t^2 - 4t + 6)\,dt =$

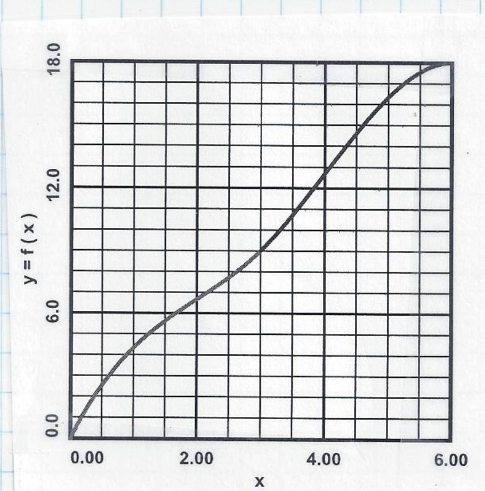
$= \left[\frac{1}{3}t^3 - 2t^2 + 6t\right]_0^x = \frac{1}{3}x^3 - 2x^2 + 6x$

$f(3) = 9$ $\quad 3 < x \le 6$: $\quad f(x) = f(3) + \int_3^x f'(t)\,dt = 9 + \int_3^x (-t^2 + 8t - 12)\,dt =$

$= 9 + \left[-\frac{1}{3}t^3 + 4t^2 - 12t\right]_3^x = 9 + \left[-\frac{1}{3}x^3 + 4x^2 - 12x - -9\right] = -\frac{1}{3}x^3 + 4x^2 - 12x + 18$

$f(x) = \begin{cases} \frac{1}{3}x^3 - 2x^2 + 6x , & 0 \le x \le 3 \\ -\frac{1}{3}x^3 + 4x^2 - 12x + 18 , & 3 < x \le 6 \end{cases}$ ◄

(c)



pg. 320 $\quad$ 6.5. Trapezoidal Rule

(12) $\int_0^2 f(x)\,dx = \int_0^2 x^2\,dx = \left[\frac{x^3}{3}\right]_0^2 = 2\frac{2}{3}$ ◄ (c)

0 $\quad$ 0.5 $\quad$ 1 $\quad$ 1.5 $\quad$ 2 $\qquad \Delta x = 0.5$

area $\approx \frac{1}{2}\left[f(0) + f(0.5)\right]\Delta x +$

$+ \frac{1}{2}\left[f(0.5) + f(1)\right]\Delta x +$

$+ \frac{1}{2}\left[f(1) + f(1.5)\right]\Delta x + \frac{1}{2}\left[f(1.5) + f(2)\right]\Delta x =$

$= \frac{1}{2}\left[f(0) + 2f(0.5) + 2f(1) + 2f(1.5) + f(2)\right]\Delta x = 2.75$ ◄ (a)

(b) Overestimation ◄

(7) $\Delta x = 1$, area $\approx \frac{1}{2}\left[f(0) + 2f(1) + 2f(2) + 2f(3) + 2f(4) + 2f(5) + f(6)\right] =$

$= \frac{1}{2}\left[12 + 20 + 18 + 22 + 26 + 32 + 18\right] = 74$ ◄

(10) $\Delta x = 200$, area $\approx \frac{1}{2}\left[0 + 2(520) + 2(800) + 2(1000) + 2(1140) + 2(1160) + \right.$

$\left. + 2(1110) + 2(860) + 0\right]\Delta x = 1,318,000\ ft^2$ $(\times 20\,ft) \Rightarrow vol \approx 26,360,000\ ft^3$

(a) ◄

(b) $(\div 1000) = 26,360$ fish $(\times 0.75) = 19,770$ $(\div 20) = 988.5$ or 988 licenses ◄

pg. 320 6.5. Trapezoidal Rule Program

(27) $I = \int_0^{\pi} \sin x\, dx = \left[-\cos x\right]_0^{\pi} = -\cos\pi + \cos 0 = -^-1 + 1 = 2$ (exact)

N	approx.	approx. − exact
10	1.983 523 538	−0.016 476 4625
100	1.999 835 504	−0.000 164 4961
1000	1.999 998 355	−0.000 001 6449

(c) −0.000 000 016 ◄

6.5. Derivation of Simpson's Rule

Supplemental

(4) $f(\xi) = a\xi^2 + b\xi + c$ $f(0) = f_i \Rightarrow c = f_i$ ◄

$f(0.5) = f_{i+0.5}$, $\frac{1}{4}a + \frac{1}{2}b + f_i = f_{i+0.5}$ $\Rightarrow a + 2b = 4f_{i+0.5} - 4f_i$ (A)

$f(1) = f_{i+1}$, $a + b + f_i = f_{i+1}$ $\Rightarrow a + b = f_{i+1} - f_i$ (B)

$(A) - (B) \Rightarrow b = 4f_{i+0.5} - 4f_i - f_{i+1} + f_i$, $b = -3f_i + 4f_{i+0.5} - f_{i+1}$ ◄

$(B) \Rightarrow a - 3f_i + 4f_{i+0.5} - f_{i+1} = f_{i+1} - f_i$, $a = 2f_i - 4f_{i+0.5} + 2f_{i+1}$ ◄

So, $f(\xi) = (2f_i - 4f_{i+0.5} + 2f_{i+1})\xi^2 + (-3f_i + 4f_{i+0.5} - f_{i+1})\xi + f_i$ ◄

pg. 320 6.5. Simpsons Rule

(16) $I = \int_1^2 \frac{dx}{x} = \left[\ln x\right]_1^2 = \ln 2 - \ln 1 = \ln 2 \approx 0.693$ ◄ (b)

HW #10

(a) 

$\Delta x = 0.5$, $f(x) = \frac{1}{x}$, $I \approx \frac{1}{6}\left[f(1) + 4f(1.25) + f(1.5)\right]\Delta x +$

$+ \frac{1}{6}\left[f(1.5) + 4f(1.75) + f(2)\right]\Delta x = \frac{1}{6}\left[f(1) + 4f(1.25) + 2f(1.5) + 4f(1.75) + f(2)\right]\Delta x$

$= 0.693\ 253\ 9683$ ◄

(29) $\Delta x = 8$, area $\approx \frac{1}{6}\left[0 + 4(18.75) + 2(24) + 4(26) + 2(24) + 4(18.75) + 0\right]\Delta x =$

$= 466\frac{2}{3}\ in^2$ ◄

Supplemental

6.5. Program for Simpson's Rule

(5) $I = \int_0^{\pi/3} \sqrt{3}\sec^2 x\, dx = \left[\sqrt{3}\tan x\right]_0^{\pi/3} = \sqrt{3}\tan\frac{\pi}{3} = 3$ ◄ (a)

N	I
2	3.016 023 936
10	3.000 042 605
100	3.000 000 004
200	3

◄ (b)

fnInt = 3 ◄ (c)