1) For $f(x) = 2x^3 - 33x^2 + 168x - 160$ defined on $x \in [\,2\,,8\,]$

a) Calculate the average value of $f(x)$ over $x \in [2\,,8]$.

b) Find the values of $c \in [2\,,8]$ which are guaranteed to exist via the Mean Value Theorem for Definite Integrals.
Hint: one of the values is $c = 8$. Find the other two values by using synthetic division and the Quadratic Formula.

2) For

$$f'(x) = \left\{ \begin{array}{lll} x^2 - 2x + 3 & , & 0 \le x \le 4 \\ x^2 - 14x + 51 & , & 4 < x \le 8 \end{array} \right\}$$

a) Is $f(x)$ differentiable at $x = 4$?

b) Calculate $y = f(x)$ subject to the initial condition $f(0) = 0$.

c) Graph $y = f(x)$.

Problems **3** through **6** concern the area under $f(x) = x \ln x$ on $x \in [1\,,9\,]$. The exact value of the area may be found by using the indefinite integral

$$\int x \ln x \; dx = \frac{x^2}{4}(2 \ln x - 1)\,.$$

3) Estimate the area using four equal subdivisions with the Trapezoidal Rule. Show your work. Also, what is the percent relative area of the estimation?

4) Estimate the area using 500 subdivisions with the Trapezoidal Rule. Also, what is the percent relative error of the estimation?

5) Estimate the area using two equal subdivisions of size $\Delta x = 4$ with Simpson's Rule. Show your work. Also, what is the percent relative error of the estimation?

6) Estimate the area using 120 subdivisions with Simpson's Rule. Also, what is the percent relative error of the estimation?