

# Quiz #10 Study Guide

10F2

(1)  $f(x) = 2x^3 - 33x^2 + 168x - 160$  on  $x \in [2, 8]$ .

(a)  $\bar{f} = \frac{1}{6} \int_2^8 f(x) dx = \frac{1}{6} \left[ \frac{1}{2}x^4 - 11x^3 + 84x^2 - 160x \right]_2^8 = 96$

(b)  $2c^3 - 33c^2 + 168c - 160 = 96$ ,  $2c^3 - 33c^2 + 168c - 256 = 0$

$c = 8$  is one value

2	-33	168	-256
16	-136	256	
2	-17	32	0

$2c^2 - 17c + 32 = 0$   $c = \frac{17 \pm \sqrt{17^2 - 4(2)(32)}}{2(2)} = \frac{17 \pm \sqrt{33}}{4}$

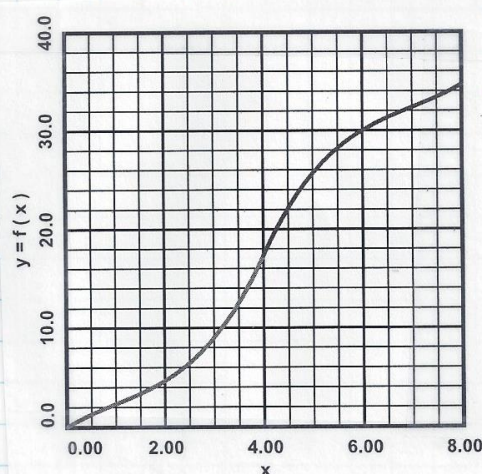
$c = 2.814$   $c = 5.686$

(2)  $f'(x) = \begin{cases} x^2 - 2x + 3, & 0 \leq x \leq 4 \\ x^2 - 14x + 51, & 4 < x \leq 8 \end{cases}$  (a)  $f'(4^-) = 11$ ,  $f'(4^+) = 11$ , so yes

(b)  $0 \leq x \leq 4$ :  $f(x) = f(0) + \int_0^x f'(t) dt = \int_0^x (t^2 - 2t + 3) dt = \left[ \frac{1}{3}t^3 - t^2 + 3t \right]_0^x = \frac{1}{3}x^3 - x^2 + 3x$ ,  $f(4) = 17\frac{1}{3}$   
 $4 < x \leq 8$ :  $f(x) = f(4) + \int_4^x f'(t) dt = 17\frac{1}{3} + \int_4^x (t^2 - 14t + 51) dt = 17\frac{1}{3} + \left[ \frac{1}{3}t^3 - 7t^2 + 51t \right]_4^x = 17\frac{1}{3} + \frac{1}{3}x^3 - 7x^2 + 51x - 113\frac{1}{3} = \frac{1}{3}x^3 - 7x^2 + 51x - 96$

$f(x) = \begin{cases} \frac{1}{3}x^3 - x^2 + 3x, & 0 \leq x \leq 4 \\ \frac{1}{3}x^3 - 7x^2 + 51x - 96, & 4 < x \leq 8 \end{cases}$

(c)

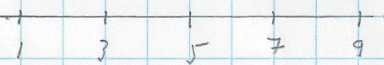


# Quiz #10 Study Guide

2 of 2

$$f(x) = x \ln x \text{ on } x \in [1, 9]$$

$$\text{exact area} = \int_1^9 x \ln x dx = \left[ \frac{x^2}{4} (2 \ln x - 1) \right]_1^9 = \frac{81}{2} \ln 9 - 20 = 68.98759538$$

(3)   $\Delta x = 2$

$$\text{est.} = \frac{1}{2} [f(1) + 2f(3) + 2f(5) + 2f(7) + f(9)] \Delta x = 69.70391614$$

$$\text{error} = 1.038\%$$

(4)  $N = 500 \Rightarrow \text{est.} = 68.98764226$   $\text{error} = 0.0000679\%$

(5)  $\Delta x = 4$

$$\text{est.} = \frac{1}{6} [f(1) + 4f(3) + 2f(5) + 4f(7) + f(9)] \Delta x = 69.02548797$$

$$\text{error} = 0.0549\%$$

(6)  $N = 120 \Rightarrow \text{est.} = 68.98759539$   $\text{error} = 0.0000000982\%$