6.3. Indefinite Integrals

When the integral sign does not have any limits on it, it is called an **indefinite integral**. Here the integral sign just means calculate the antiderivative.

$$\boxed{\int f'(x)\,dx = f(x) + k} \quad (k \equiv \text{arbitrary constant})$$

$$\frac{d}{dx}\int f'(x)\,dx = f'(x) \quad \text{or} \quad \boxed{\frac{d}{dx}\int f(x)\,dx = f(x)}$$

Example #1. Calculate the indefinite integrals

(a) $\int \sec^2 x\,dx$ (b) $\int (4x^2 - 9x + 7)\,dx$ (c) $\int \left(14\sqrt[5]{x^3} + \frac{7}{\sqrt[3]{x^8}}\right)dx$

SOLUTION:

(a) $f'(x) = \sec^2 x$, $f(x) = \tan x \Rightarrow \int \sec^2 x\,dx = \tan x + k$

(b) $\int (4x^2 - 9x + 7)\,dx = 4\frac{x^3}{3} - 9\frac{x^2}{2} + 7x + k = \frac{4}{3}x^3 - \frac{9}{2}x^2 + 7x + k$

(c) $f'(x) = 14x^{3/5} + 7x^{-8/3}$ $f(x) = 14\frac{x^{8/5}}{8/5} + 7\frac{x^{-5/3}}{-5/3} = \frac{35}{4}x^{8/5} - \frac{21}{5}x^{-5/3}$

$$\int \left(14\sqrt[5]{x^3} + \frac{7}{\sqrt[3]{x^8}}\right)dx = \frac{35\sqrt[5]{x^8}}{4} - \frac{21}{5\sqrt[3]{x^5}} + k$$

Recall the hyperbolic sine and cosine functions...

$\sinh x = \frac{1}{2}(e^x - e^{-x})$, $\cosh x = \frac{1}{2}(e^x + e^{-x})$, $\frac{d\sinh x}{dx} = \cosh x$, $\frac{d\cosh x}{dx} = \sinh x$

Example #2, verify the indefinite integrals by differentiation.

(a) $\int \sin^2 x\,dx = \frac{1}{2}\left[x - \sin x\cos x\right] + k$

(b) $\int x^2\cosh x\,dx = x^2\sinh x - 2x\cosh x + 2\sinh x + k$

SOLUTIONS:

6.3. Indefinite Integrals

(a) $f(x)=\frac{1}{2}\left[x-\sin x\cos x\right]+k$, $f'(x)=\frac{1}{2}\left[1-\left(\cos x\cos x+\sin x\cdot -\sin x\right)\right]=$

$=\frac{1}{2}\left[1-\cos^2 x+\sin^2 x\right]=\frac{1}{2}\left[\sin^2 x+\sin^2 x\right]=\sin^2 x$ ◄

(b) $f(x)=x^2\sinh x-2x\cosh x+2\sinh x+k$,

$f'(x)=2x\sinh x+x^2\cosh x-2\left(1\cdot\cosh x+x\sinh x\right)+2\cosh x=$

$=2x\sinh x+x^2\cosh x-2\cosh x-2x\sinh x+2\cosh x=x^2\cosh x$ ◄

CLASS WORK

(1) Calculate the indefinite integrals

(a) $\int \sec x\tan x\,dx$ (b) $\int\left(6x^2+8x-11\right)dx$ (c) $\int\left(4\sqrt{x^7}-\frac{11}{\sqrt{x^5}}\right)dx$

(2) Verify the indefinite integrals by differentiation

(a) $\int\cos^2 x\,dx=\frac{1}{2}\left[x+\sin x\cos x\right]+k$

(b) $\int x^2\sin x\,dx=-x^2\cos x+2x\sin x+2\cos x+k$

SOLUTIONS

(1) (a) $f'(x)=\sec x\tan x$, $f(x)=\sec x \Rightarrow \int\sec x\tan x\,dx=\sec x+k$ ◄

(b) $\int\left(6x^2+8x-11\right)dx=6\cdot\frac{x^3}{3}+8\cdot\frac{x^2}{2}-11x+k=2x^3+4x^2-11x+k$ ◄

(c) $f'(x)=4x^{7/2}-11x^{-5/2}$, $f(x)=4\,\frac{x^{9/2}}{9/2}-11\,\frac{x^{-3/2}}{-3/2}+k=$

$=\frac{8}{9}x^{9/2}+\frac{22}{3}x^{-3/2}+k \Rightarrow \int\left(4\sqrt{x^7}-\frac{11}{\sqrt{x^5}}\right)dx=\frac{8\sqrt{x^9}}{9}+\frac{22}{3\sqrt{x^3}}+k$ ◄

(2) (a) $f'(x)=\frac{1}{2}\left[1+\left(\cos x\cos x+\sin x\cdot-\sin x\right)\right]=\frac{1}{2}\left[1+\cos^2 x-\sin^2 x\right]=$

$=\frac{1}{2}\left[\cos^2 x+\cos^2 x\right]=\cos^2 x$ ◄

(b) $f'(x) = -(2x\cos x + x^2 \cdot -\sin x) + 2(1\cdot\sin x + x\cos x) + 2\cdot -\sin x =$

$= -\cancel{2x\cos x} + x^2\sin x + 2\sin x + \cancel{2x\cos x} - 2\sin x = x^2\sin x$