6.4. Leibniz's Rule

FTC. 1 is $\int_a^b f'(t)\,dt = f(b) - f(a)$. Let $a = a(x)$ and $b = b(x)$

$$\int_{a(x)}^{b(x)} f'(t)\,dt = f[b(x)] - f[a(x)], \text{ differentiate} \Rightarrow$$

$$\frac{d}{dx}\int_{a(x)}^{b(x)} f'(t)\,dt = \frac{df}{db}\frac{db}{dx} - \frac{df}{da}\frac{da}{dx} = f'(b)\frac{db}{dx} - f'(a)\frac{da}{dx} \text{ or}$$

$$\boxed{\frac{d}{dx}\int_{a(x)}^{b(x)} f(t)\,dt = f(b)\frac{db}{dx} - f(a)\frac{da}{dx}}$$ ← Leibniz's Rule

Example: Calculate $\frac{d}{dx}\int_{x^2}^{x^3} \sin t\,dt$

SOLUTION:

$$\frac{d}{dx}\int_{x^2}^{x^3} \sin t\,dt = \sin(x^3)\cdot 3x^2 - \sin(x^2)\cdot 2x = 3x^2\sin(x^3) - 2x\sin(x^2)$$

CLASS WORK

Calculate $\frac{d}{dx}\int_{\sin x}^{\cos x} t^2\,dt$

SOLUTION

$$\frac{d}{dx}\int_{\sin x}^{\cos x} t^2\,dt = \cos^2 x\cdot -\sin x - \sin^2 x\cdot\cos x = -\sin x\cos x(\sin x + \cos x)$$