6.1. Estimating Area Under Curves

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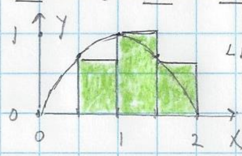
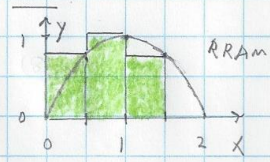
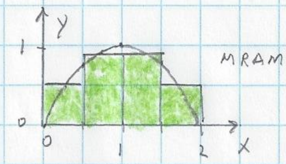
(7) 

(8) 



$f(x) = 2x - x^2$

$\Delta x = 0.5$

$[f(0) + f(0.5) + f(1) + f(1.5)]\Delta x$

$= 1.25$ ◄

$[f(0.5) + f(1) + f(1.5) + f(2)]\Delta x = 1.25$ ◄

$[f(0.25) + f(0.75) + f(1.25) + f(1.75)]\Delta x$

$= 1.375$ ◄

(22) Each rectangle is a cylinder with volume $\pi r^2 \Delta x = \pi\left(\sqrt{16-x^2}\right)^2 \Delta x =$
$= \pi(16-x^2)\Delta x = \pi f(x)\Delta x$, i.e., $f(x) = 16 - x^2$, $\Delta x = 0.5$

vol $\approx \pi\,[f(0) + f(0.5) + f(1) + f(1.5) + f(2) + f(2.5) + f(3) + f(3.5)]\Delta x = 46.5\pi$ ◄

exact $= \frac{2}{3}\pi r^3 = \frac{2}{3}\pi(4)^3 = 42\frac{2}{3}\pi$ error $= \dfrac{46.5\pi - 42\frac{2}{3}\pi}{42\frac{2}{3}\pi} = 8.98\%$ ◄

(23) vol $\approx \pi\,[f(0.5) + f(1) + f(1.5) + f(2) + f(2.5) + f(3) + f(3.5) + f(4)]\Delta x = 38.5\pi$ ◄

error $= \dfrac{38.5\pi - 42\frac{2}{3}\pi}{42\frac{2}{3}\pi} = -9.77\%$ ◄

6.1. Program for RAM

pg. 277

(9) $f(x) = 2x - x^2$

N	LRAM	MRAM	RRAM
10	1.32	1.34	1.32
50	1.3328	1.3336	1.3328
100	1.3332	1.3334	1.3332
500	1.333328	1.333336	1.333328

(10) The exact area is $1\frac{1}{3}$ ◄

(15, 16) Same as problems 22 & 23 except over $x \in [-4, 4]$.

N	MRAM	% rel. error
10	85.76π	0.5
20	85.44π	0.125
40	85.36π	0.03125
80	85.34π	0.00781
160	85.335π	0.00195

exact is $\frac{4}{3}\pi r^3 = \frac{4}{3}\pi \cdot 4^3 = 85\frac{1}{3}\pi$

HW #9

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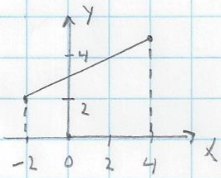
6.2. The Definite Integral

(7) $\int_{-2}^{1} 5\,dx = (1--2)(5) = 15$

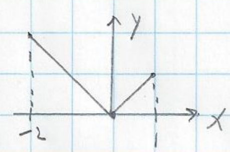
(11) $\int_{-2.1}^{3.4} 0.5\,ds = (3.4--2.1)(0.5) = 2.75$

(12) $\int_{\sqrt{2}}^{\sqrt{18}} \sqrt{2}\,dr = (\sqrt{18}-\sqrt{2})\sqrt{2} = \sqrt{36}-2 = 6-2 = 4$

(13)

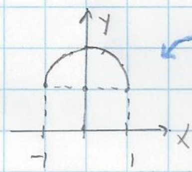


$\int_{-2}^{4} \left(\frac{x}{2}+3\right)dx = \frac{1}{2}(b_1+b_2)h = \frac{1}{2}(2+5)(6) = 21$



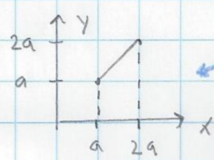
(17) $\int_{-2}^{1} |x|\,dx = \frac{1}{2}b_1h_1 + \frac{1}{2}b_2h_2 = \frac{1}{2}(2)(2) + \frac{1}{2}(1)(1) = 2.5$

(20)



$\int_{-1}^{1} \left(1+\sqrt{1-x^2}\right)dx = bh + \frac{1}{2}\pi r^2 = (2)(1) + \frac{1}{2}\pi\cdot 1^2 = 2 + \frac{\pi}{2}$

(27)



$\int_{a}^{2a} x\,dx = \frac{1}{2}(b_1+b_2)h = \frac{1}{2}(a+2a)(a) = \frac{3}{2}a^2$

6.2 & 6.3. Negative Areas & Properties of Integrals

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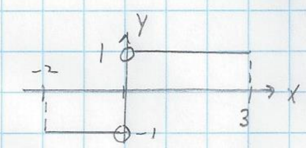
(41)



Discontinuous at $x=0$

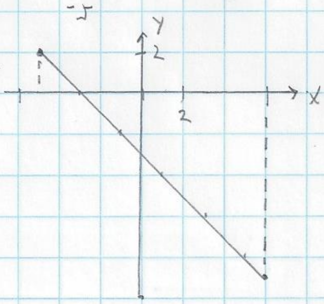
$\int_{-2}^{3} \frac{x}{|x|}\,dx = -b_1h_1 + b_2h_2 = -(2)(1) + (3)(1) = 1$

(44) $\int_{-5}^{6} \frac{9-x^2}{x-3}\,dx = \int_{-5}^{6} \frac{(3-x)(3+x)}{-(3-x)}\,dx = \int_{-5}^{6} (-x-3)\,dx = \frac{1}{2}b_1h_1 - \frac{1}{2}b_2h_2 =$

$= \frac{1}{2}(2)(2) - \frac{1}{2}(9)(9) =$

$= 2 - 40.5 = -38.5$

Discontinuous (hole) at $x=3$



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(1) $\int_1^2 f(x)\,dx = -4, \quad \int_1^5 f(x)\,dx = 6, \quad \int_1^5 g(x)\,dx = 8.$

(a) $\int_2^2 g(x)\,dx = 0$ (b) $\int_5^1 g(x)\,dx = -\int_1^5 g(x)\,dx = -8$

(c) $\int_1^2 3f(x)\,dx = 3\int_1^2 f(x)\,dx = 3(-4) = -12$

(d) $\int_1^5 f(x)\,dx = \int_1^2 f(x)\,dx + \int_2^5 f(x)\,dX, \quad 6 = -4 + \int_2^5 f(x)\,dx, \quad \int_2^5 f(x)\,dx = 10$

(e) $\int_1^5 [f(x) - g(x)]\,dx = \int_1^5 f(x)\,dx - \int_1^5 g(x)\,dx = 6 - 8 = -2$

(f) $\int_1^5 [4f(x) - g(x)]\,dx = \int_1^5 4f(x)\,dx - \int_1^5 g(x)\,dx = 4\int_1^5 f(x)\,dx - \int_1^5 g(x)\,dx = 4(6) - 8 = 16$

6.4. The Fundamental Theorem of Calculus (FTC)

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(27) $\int_{1/2}^3 \left(2 - \frac{1}{x}\right)dx = f(3) - f(\tfrac{1}{2}) = [2(3) - \ln 3] - [2(\tfrac{1}{2}) - \ln(\tfrac{1}{2})] = 6 - \ln 3 - 1 + \ln 2^{-1} =$

$= 5 - \ln 3 - \ln 2 = 5 - \ln 6 \approx 3.208$

$f'(x) = 2 - \frac{1}{x}, \quad f(x) = 2x - \ln x$

(28) $\int_2^{-1} 3^x\,dx = f(-1) - f(2) = \frac{1}{3\ln 3} - \frac{9}{\ln 3} = -\frac{26}{3\ln 3} \approx -7.889$

$f'(x) = 3^x, \quad f(x) = \frac{3^x}{\ln 3}$

(29) $\int_0^1 (x^2 + \sqrt{x})\,dx = f(1) - f(0) = 1$

$f'(x) = x^2 + x^{1/2}, \quad f(x) = \frac{x^3}{3} + \frac{x^{3/2}}{3/2} = \frac{1}{3}x^3 + \frac{2}{3}\sqrt{x^3}$

(35) $\int_0^{\pi/3} 2\sec^2\theta\,d\theta = 2\int_0^{\pi/3} \sec^2\theta\,d\theta = 2\,[f(\pi/3) - f(0)] = 2\,[\sqrt{3} - 0] = 2\sqrt{3} \approx 3.464$

$f'(\theta) = \sec^2\theta, \quad f(\theta) = \tan\theta$

(40) $\int_1^4 \frac{1 - \sqrt{u}}{\sqrt{u}}\,du = \int_1^4 \left(\frac{1}{\sqrt{u}} - 1\right)du = f(4) - f(1) = 0 - 1 = -1$

$f'(u) = u^{-1/2} - 1, \quad f(u) = \frac{u^{1/2}}{1/2} - u = 2\sqrt{u} - u$

HW #9

(45) $A = \int_0^1 x^2\,dx + \int_1^2 (2-x)\,dx = \left[\frac{x^3}{3}\right]_0^1 + \left[2x - \frac{x^2}{2}\right]_1^2 = \frac{1}{3} - 0 + 2 - \frac{3}{2} = \frac{5}{6}$ ◄

(47) $A = 2\pi - \int_0^{\pi} (1+\cos x)\,dx = 2\pi - \left[x + \sin x\right]_0^{\pi} = 2\pi - [\pi - 0] = \pi$ ◄

6.3. Indefinite Integrals

Supplemental

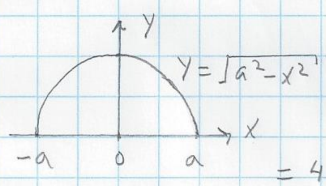
(1) $\int f'(x)\,dx = f(x) \qquad f(x) = \frac{1}{2}x\sqrt{a^2-x^2} + \frac{a^2}{2}\sin^{-1}\left(\frac{x}{a}\right) + k,$

$$f'(x) = \frac{1}{2}\left[1\cdot\sqrt{a^2-x^2}\cdot\frac{\sqrt{a^2-x^2}}{\sqrt{a^2-x^2}} + x\cdot\frac{1}{2}\frac{1}{\sqrt{a^2-x^2}}\cdot -2x\right]$$

$$+\frac{a^2}{2}\frac{1}{\sqrt{1-\left(\frac{x}{a}\right)^2}}\cdot\frac{1}{a} = \frac{1}{2}\left[\frac{a^2-x^2-x^2}{\sqrt{a^2-x^2}}\right] + \frac{a}{2\sqrt{\frac{a^2}{a^2}-\frac{x^2}{a^2}}} =$$

$$= \frac{a^2-2x^2}{2\sqrt{a^2-x^2}} + \frac{a}{\frac{2}{a}\sqrt{a^2-x^2}} = \frac{a^2-2x^2+a^2}{2\sqrt{a^2-x^2}} = \frac{2(a^2-x^2)}{2\sqrt{a^2-x^2}} = \sqrt{a^2-x^2}$$ ◄

(2)



$$A_{cir} = 4\int_0^a \sqrt{a^2-x^2}\,dx$$

$$= 4\left[\frac{1}{2}x\sqrt{a^2-x^2} + \frac{a^2}{2}\sin^{-1}\left(\frac{x}{a}\right)\right]_0^a =$$

$$= 4\left[\frac{a^2}{2}\sin^{-1}(1) - \frac{a^2}{2}\sin^{-1}(0)\right] = 4\left[\frac{a^2}{2}\cdot\frac{\pi}{2} - 0\right] = 4\cdot\frac{\pi a^2}{4} = \pi a^2$$ ◄

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6.4 Leibniz's Rule

$$\frac{d}{dx}\int_a^b f(t)\,dt = f(b)\frac{db}{dx} - f(a)\frac{da}{dx}$$

(1) $\frac{d}{dx}\int_0^x \sin^2 t\,dt = \sin^2 x\cdot 1 = \sin^2 x$ ◄

(5) $\frac{d}{dx}\int_2^x \tan^3 u\,du = \tan^3 x\cdot 1 = \tan^3 x$ ◄

HW # 9

(9) $\frac{d}{dx}\int_0^{x^2} e^{t^2}\,dt = e^{x^4}\cdot 2x = 2xe^{x^4}$

(13) $\frac{d}{dx}\int_x^6 \ln(1+t^2)\,dt = 0 - \ln(1+x^2)\cdot 1 = -\ln(1+x^2) = \ln\left(\frac{1}{1+x^2}\right)$

(15) $\frac{d}{dx}\int_{x^3}^5 \frac{\cos t}{t^2+2}\,dt = 0 - \frac{\cos x^3}{x^6+2}\cdot 3x^2 = -\frac{3x^2\cos x^3}{x^6+2}$

(19) $\int_{x^2}^{x^3} \cos 2t\,dt = \cos 2x^3\cdot 3x^2 - \cos 2x^2\cdot 2x = 3x^2\cos 2x^3 - 2x\cos 2x^2$

$y = \int_a^x f(t)\,dt + C$, $\frac{dy}{dx} = f(x)$

(21) $\frac{dy}{dx} = \sin^3 x \quad (x,y) = (5,0) \quad y = \int_5^x \sin^3 t\,dt + C$, $0 = \int_5^5 \sin^3 t\,dt + C = 0 + C$,

$C = 0 \Rightarrow y = \int_5^x \sin^3 t\,dt$

(23) $\frac{dy}{dx} = \ln(\sin x + 5) \quad (x,y) = (2,3) \quad y = \int_2^x \ln(\sin t + 5)\,dt + C$,

$3 = \int_2^2 \ln(\sin t + 5)\,dt + C = 0 + C$, $C = 3 \Rightarrow y = 3 + \int_2^x \ln(\sin t + 5)\,dt$