Section 7.1 Exercises

In Exercises 1–10, find the general solution to the exact differential equation.

1. $\frac{dy}{dx} = 5x^4 - \sec^2 x$

2. $\frac{dy}{dx} = \sec x \tan x - e^x$

3. $\frac{dy}{dx} = \sin x - e^{-x} + 8x^3$

4. $\frac{dy}{dx} = \frac{1}{x} - \frac{1}{x^2}\ (x > 0)$

5. $\frac{dy}{dx} = 5^x \ln 5 + \frac{1}{x^2+1}$

6. $\frac{dy}{dx} = \frac{1}{\sqrt{1-x^2}} - \frac{1}{\sqrt{x}}$

7. $\frac{dy}{dt} = 3t^2 \cos(t^3)$

8. $\frac{dy}{dt} = (\cos t)\, e^{\sin t}$

9. $\frac{du}{dx} = (\sec^2 x^5)(5x^4)$

10. $\frac{dy}{du} = 4(\sin u)^3(\cos u)$

In Exercises 11–20, solve the initial value problem explicitly.

11. $\frac{dy}{dx} = 3\sin x$ and $y = 2$ when $x = 0$

12. $\frac{dy}{dx} = 2e^x - \cos x$ and $y = 3$ when $x = 0$

13. $\frac{du}{dx} = 7x^6 - 3x^2 + 5$ and $u = 1$ when $x = 1$

14. $\frac{dA}{dx} = 10x^9 + 5x^4 - 2x + 4$ and $A = 6$ when $x = 1$

15. $\frac{dy}{dx} = -\frac{1}{x^2} - \frac{3}{x^4} + 12$ and $y = 3$ when $x = 1$

16. $\frac{dy}{dx} = 5\sec^2 x - \frac{3}{2}\sqrt{x}$ and $y = 7$ when $x = 0$

17. $\frac{dy}{dt} = \frac{1}{1+t^2} + 2^t \ln 2$ and $y = 3$ when $t = 0$

18. $\frac{dx}{dt} = \frac{1}{t} - \frac{1}{t^2} + 6$ and $x = 0$ when $t = 1$

19. $\frac{dv}{dt} = 4\sec t \tan t + e^t + 6t$ and $v = 5$ when $t = 0$

20. $\frac{ds}{dt} = t(3t - 2)$ and $s = 0$ when $t = 1$

In Exercises 21–24, solve the initial value problem using the Fundamental Theorem. (Your answer will contain a definite integral.)

21. $\frac{dy}{dx} = \sin(x^2)$ and $y = 5$ when $x = 1$

22. $\frac{du}{dx} = \sqrt{2 + \cos x}$ and $u = -3$ when $x = 0$

23. $F'(x) = e^{\cos x}$ and $F(2) = 9$

24. $G'(s) = \sqrt[3]{\tan s}$ and $G(0) = 4$

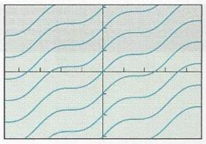
In Exercises 25–28, match the differential equation with the graph of a family of functions (a)–(d) at the top of the next page that solve it. Use slope analysis, not your graphing calculator.

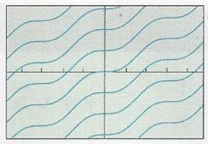
25. $\frac{dy}{dx} = (\sin x)^2$

26. $\frac{dy}{dx} = (\sin x)^3$

27. $\frac{dy}{dx} = (\cos x)^2$

28. $\frac{dy}{dx} = (\cos x)^3$


(a)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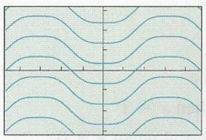
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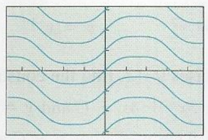
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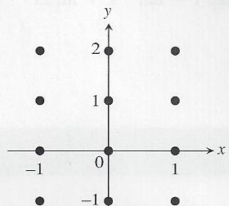
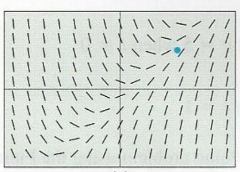
In Exercises 29–34, construct a slope field for the differential equation. In each case, copy the graph at the right and draw tiny segments through the twelve lattice points shown in the graph. Use slope analysis, not your graphing calculator.

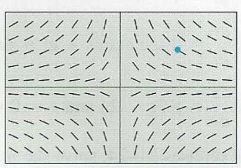


29. $\dfrac{dy}{dx} = x$ **30.** $\dfrac{dy}{dx} = y$ **31.** $\dfrac{dy}{dx} = 2x + y$

32. $\dfrac{dy}{dx} = 2x - y$ **33.** $\dfrac{dy}{dx} = x + 2y$ **34.** $\dfrac{dy}{dx} = x - 2y$

In Exercises 35–40, match the differential equation with the appropriate slope field. Then use the slope field to sketch the graph of the particular solution through the highlighted point $(3, 2)$. (All slope fields are shown in the window $[-6, 6]$ by $[-4, 4]$.)


(a)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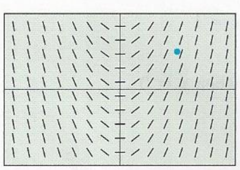
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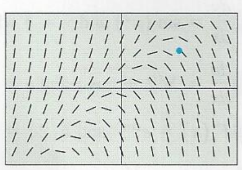
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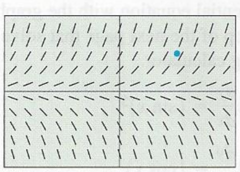
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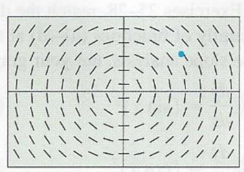
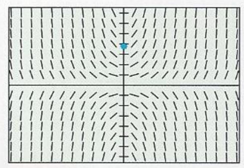

(f)

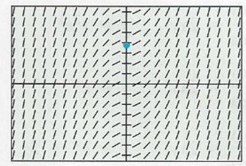
35. $\dfrac{dy}{dx} = x$ **36.** $\dfrac{dy}{dx} = y$

37. $\dfrac{dy}{dx} = x - y$ **38.** $\dfrac{dy}{dx} = y - x$

39. $\dfrac{dy}{dx} = -\dfrac{y}{x}$ **40.** $\dfrac{dy}{dx} = -\dfrac{x}{y}$

In Exercises 41–46, match the differential equation with the appropriate slope field. Then use the slope field to sketch the graph of the particular solution through the highlighted point $(0, 2)$. (All slope fields are shown in the window $[-6, 6]$ by $[-4, 4]$.)


(a)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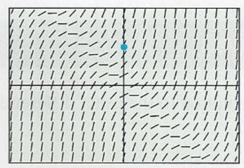
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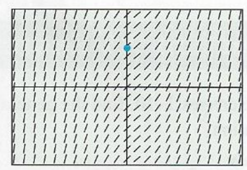
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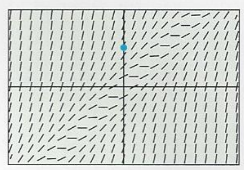
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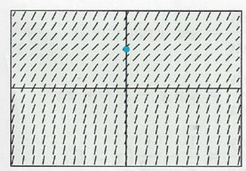

(f)

41. $\dfrac{dy}{dx} = \sqrt{x^2 - x + 1}$ **42.** $\dfrac{dy}{dx} = \sqrt{y^2 - 4y + 5}$

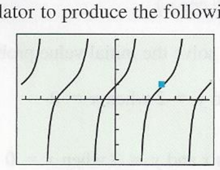
43. $\dfrac{dy}{dx} = |x + y|$ **44.** $\dfrac{dy}{dx} = |x - y|$

45. $\dfrac{dy}{dx} = |x|$ **46.** $\dfrac{dy}{dx} = xy$

47. **(a)** Sketch a graph of the solution to the initial value problem

$$\frac{dy}{dx} = \sec^2 x \text{ and } y = 1 \text{ when } x = \pi.$$

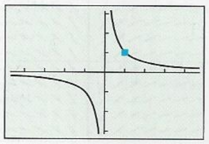
(b) **Writing to Learn** A student solved part (a) and used a graphing calculator to produce the following graph:


$[-2\pi, 2\pi]$ by $[-4, 4]$

How would you explain to this student why this graph is *not* the correct answer to part (a)?

48. **(a)** Sketch a graph of the solution to the initial value problem $\dfrac{dy}{dx} = -x^{-2}$ and $y = 1$ when $x = 1$.

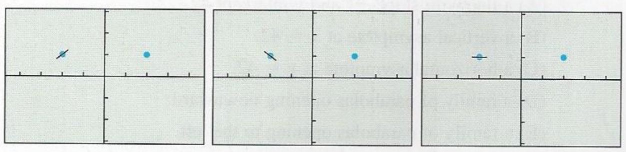
(b) Writing to Learn A student solved part (a) and used a graphing calculator to produce the following graph:



[−4.7, 4.7] by [−3.1, 3.1]

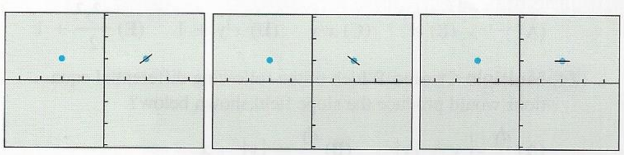
How would you explain to this student why this graph is *not* the correct answer to part (a)?

49. ***Left Field Line*** A single line from the slope field for $\dfrac{dy}{dx} = 2y + x$ is shown in the second quadrant of one of the following three graphs. Choose the only possible graph and draw a line for the same slope field through the reflected point in the first quadrant. All graphs are shown in the window $[-4.7, 4.7]$ by $[-3.1, 3.1]$.



(a) (b) (c)

50. ***Right Field Line*** A single line from the slope field for $\dfrac{dy}{dx} = y^2 - x$ is shown in the first quadrant of one of the following three graphs. Choose the only possible graph and draw a line for the same slope field through the reflected point in the second quadrant. All graphs are shown in the window $[-4.7, 4.7]$ by $[-3.1, 3.1]$.



(a) (b) (c)

In Exercises 51–54, use Euler's Method with increments of $\Delta x = 0.1$ to approximate the value of y when $x = 1.3$.

51. $\dfrac{dy}{dx} = x - 1$ and $y = 2$ when $x = 1$

52. $\dfrac{dy}{dx} = y - 1$ and $y = 3$ when $x = 1$

53. $\dfrac{dy}{dx} = y - x$ and $y = 2$ when $x = 1$

54. $\dfrac{dy}{dx} = 2x - y$ and $y = 0$ when $x = 1$

In Exercises 55–58, use Euler's Method with increments of $\Delta x = -0.1$ to approximate the value of y when $x = 1.7$.

55. $\dfrac{dy}{dx} = 2 - x$ and $y = 1$ when $x = 2$

56. $\dfrac{dy}{dx} = 1 + y$ and $y = 0$ when $x = 2$

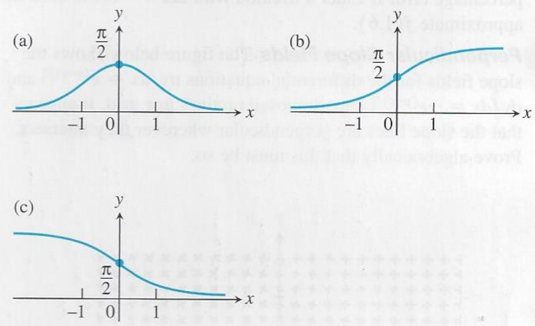
57. $\dfrac{dy}{dx} = x - y$ and $y = 2$ when $x = 2$

58. $\dfrac{dy}{dx} = x - 2y$ and $y = 1$ when $x = 2$

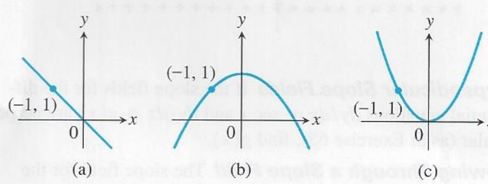
In Exercises 59 and 60, **(a)** determine which graph shows the solution of the initial value problem without actually solving the problem.

(b) Writing to Learn Explain how you eliminated two of the possibilities.

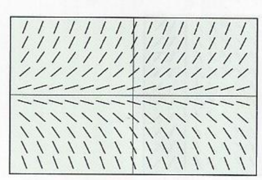
59. $\dfrac{dy}{dx} = \dfrac{1}{1 + x^2}, \quad y(0) = \dfrac{\pi}{2}$



60. $\dfrac{dy}{dx} = -x, \quad y(-1) = 1$

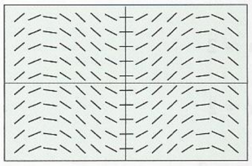


61. Writing to Learn Explain why $y = x^2$ could not be a solution to the differential equation with slope field shown below.



[−4.7, 4.7] by [−3.1, 3.1]

62. Writing to Learn Explain why $y = \sin x$ could not be a solution to the differential equation with slope field shown below.

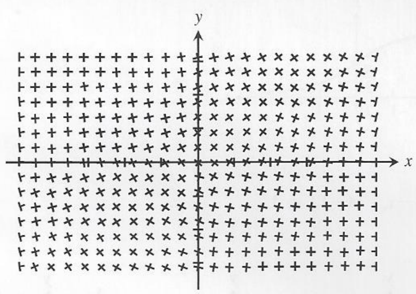


$[-4.7, 4.7]$ by $[-3.1, 3.1]$

63. *Percentage Error* Let $y = f(x)$ be the solution to the initial value problem $dy/dx = 2x + 1$ such that $f(1) = 3$. Find the percentage error if Euler's Method with $\Delta x = 0.1$ is used to approximate $f(1.4)$.

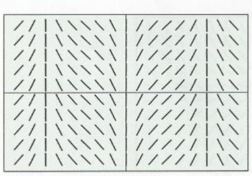
64. *Percentage Error* Let $y = f(x)$ be the solution to the initial value problem $dy/dx = 2x - 1$ such that $f(2) = 3$. Find the percentage error if Euler's Method with $\Delta x = -0.1$ is used to approximate $f(1.6)$.

65. *Perpendicular Slope Fields* The figure below shows the slope fields for the differential equations $dy/dx = e^{(x-y)/2}$ and $dy/dx = -e^{(y-x)/2}$ superimposed on the same grid. It appears that the slope lines are perpendicular wherever they intersect. Prove algebraically that this must be so.



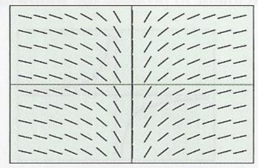
66. *Perpendicular Slope Fields* If the slope fields for the differential equations $dy/dx = \sec x$ and $dy/dx = g(x)$ are perpendicular (as in Exercise 65), find $g(x)$.

67. *Plowing Through a Slope Field* The slope field for the differential equation $dy/dx = \csc x$ is shown below. Find a function that will be *perpendicular* to every line it crosses in the slope field. [*Hint:* First find a differential equation that will produce a perpendicular slope field.]



$[-4.7, 4.7]$ by $[-3.1, 3.1]$

68. *Plowing Through a Slope Field* The slope field for the differential equation $dy/dx = 1/x$ is shown below. Find a function that will be *perpendicular* to every line it crosses in the slope field. [*Hint:* First find a differential equation that will produce a perpendicular slope field.]



$[-4.7, 4.7]$ by $[-3.1, 3.1]$

Standardized Test Questions

69. True or False Any two solutions to the differential equation $dy/dx = 5$ are parallel lines. Justify your answer.

70. True or False If $f(x)$ is a solution to $dy/dx = 2x$, then $f^{-1}(x)$ is a solution to $dy/dx = 2y$. Justify your answer.

71. Multiple Choice A slope field for the differential equation $dy/dx = 42 - y$ will show

(A) a line with slope -1 and y-intercept 42.

(B) a vertical asymptote at $x = 42$.

(C) a horizontal asymptote at $y = 42$.

(D) a family of parabolas opening downward.

(E) a family of parabolas opening to the left.

72. Multiple Choice For which of the following differential equations will a slope field show nothing but negative slopes in the fourth quadrant?

(A) $\dfrac{dy}{dx} = -\dfrac{x}{y}$ **(B)** $\dfrac{dy}{dx} = xy + 5$ **(C)** $\dfrac{dy}{dx} = xy^2 - 2$

(D) $\dfrac{dy}{dx} = \dfrac{x^3}{x^2}$ **(E)** $\dfrac{dy}{dx} = \dfrac{y}{x^2} - 3$

73. Multiple Choice If $dy/dx = 2xy$ and $y = 1$ when $x = 0$, then $y =$

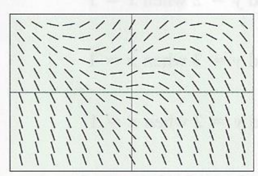
(A) y^{2x} **(B)** e^{x^2} **(C)** x^2y **(D)** $x^2y + 1$ **(E)** $\dfrac{x^2y^2}{2} + 1$

74. Multiple Choice Which of the following differential equations would produce the slope field shown below?

(A) $\dfrac{dy}{dx} = y - |x|$ **(B)** $\dfrac{dy}{dx} = |y| - x$

(C) $\dfrac{dy}{dx} = |y - x|$ **(D)** $\dfrac{dy}{dx} = |y + x|$

(E) $\dfrac{dy}{dx} = |y| - |x|$



$[-3, 3]$ by $[-1.98, 1.98]$

Explorations

75. *Solving Differential Equations* Let $\dfrac{dy}{dx} = x - \dfrac{1}{x^2}$.

(a) Find a solution to the differential equation in the interval $(0, \infty)$ that satisfies $y(1) = 2$.

(b) Find a solution to the differential equation in the interval $(-\infty, 0)$ that satisfies $y(-1) = 1$.

(c) Show that the following piecewise function is a solution to the differential equation for any values of C_1 and C_2.

$$y = \begin{cases} \dfrac{1}{x} + \dfrac{x^2}{2} + C_1 & x < 0 \\ \dfrac{1}{x} + \dfrac{x^2}{2} + C_2 & x > 0 \end{cases}$$

(d) Choose values for C_1 and C_2 so that the solution in part (c) agrees with the solutions in parts (a) and (b).

(e) Choose values for C_1 and C_2 so that the solution in part (c) satisfies $y(2) = -1$ and $y(-2) = 2$.

76. *Solving Differential Equations* Let $\dfrac{dy}{dx} = \dfrac{1}{x}$.

(a) Show that $y = \ln x + C$ is a solution to the differential equation in the interval $(0, \infty)$.

(b) Show that $y = \ln(-x) + C$ is a solution to the differential equation in the interval $(-\infty, 0)$.

(c) Writing to Learn Explain why $y = \ln|x| + C$ is a solution to the differential equation in the domain $(-\infty, 0) \cup (0, \infty)$.

(d) Show that the function

$$y = \begin{cases} \ln(-x) + C_1, & x < 0 \\ \ln x + C_2, & x > 0 \end{cases}$$

is a solution to the differential equation for any values of C_1 and C_2.

Extending the Ideas

77. *Second-Order Differential Equations* Find the general solution to each of the following second-order differential equations by first finding dy/dx and then finding y. The general solution will have two unknown constants.

(a) $\dfrac{d^2y}{dx^2} = 12x + 4$ **(b)** $\dfrac{d^2y}{dx^2} = e^x + \sin x$

(c) $\dfrac{d^2y}{dx^2} = x^3 + x^{-3}$

78. *Second-Order Differential Equations* Find the specific solution to each of the following second-order initial value problems by first finding dy/dx and then finding y.

(a) $\dfrac{d^2y}{dx^2} = 24x^2 - 10$. When $x = 1$, $\dfrac{dy}{dx} = 3$ and $y = 5$.

(b) $\dfrac{d^2y}{dx^2} = \cos x - \sin x$. When $x = 0$, $\dfrac{dy}{dx} = 2$ and $y = 0$

(c) $\dfrac{d^2y}{dx^2} = e^x - x$. When $x = 0$, $\dfrac{dy}{dx} = 0$ and $y = 1$.

79. *Differential Equation Potpourri* For each of the following differential equations, find at least one particular solution. You will need to call on past experience with functions you have differentiated. For a greater challenge, find the general solution.

(a) $y' = x$ **(b)** $y' = -x$ **(c)** $y' = y$

(d) $y' = -y$ **(e)** $y' = xy$

80. *Second-Order Potpourri* For each of the following second-order differential equations, find at least one particular solution. You will need to call on past experience with functions you have differentiated. For a significantly greater challenge, find the general solution (which will involve two unknown constants).

(a) $y'' = x$ **(b)** $y'' = -x$ **(c)** $y'' = -\sin x$

(d) $y'' = y$ **(e)** $y'' = -y$