Section 7.2 Exercises

In Exercises 1–6, find the indefinite integral.

1. $\int (\cos x - 3x^2)\, dx$
2. $\int x^{-2}\, dx$
3. $\int \left(t^2 - \frac{1}{t^2}\right) dt$
4. $\int \frac{dt}{t^2 + 1}$
5. $\int (3x^4 - 2x^{-3} + \sec^2 x)\, dx$
6. $\int (2e^x + \sec x \tan x - \sqrt{x})\, dx$

In Exercises 7–12, use differentiation to verify the antiderivative formula.

7. $\int \csc^2 u\, du = -\cot u + C$
8. $\int \csc u \cot u = -\csc u + C$
9. $\int e^{2x}\, dx = \frac{1}{2} e^{2x} + C$
10. $\int 5^x\, dx = \frac{1}{\ln 5} 5^x + C$
11. $\int \frac{1}{1 + u^2}\, du = \tan^{-1} u + C$
12. $\int \frac{1}{\sqrt{1 - u^2}}\, du = \sin^{-1} u + C$

In Exercises 13–16, verify that $\int f(u)\, du \neq \int f(u)\, dx$.

13. $f(u) = \sqrt{u}$ and $u = x^2$ $(x > 0)$
14. $f(u) = u^2$ and $u = x^5$
15. $f(u) = e^u$ and $u = 7x$
16. $f(u) = \sin u$ and $u = 4x$

In Exercises 17–24, use the indicated substitution to evaluate the integral. Confirm your answer by differentiation.

17. $\int \sin 3x\, dx, \quad u = 3x$
18. $\int x \cos (2x^2)\, dx, \quad u = 2x^2$
19. $\int \sec 2x \tan 2x\, dx, \quad u = 2x$
20. $\int 28(7x - 2)^3\, dx, \quad u = 7x - 2$
21. $\int \frac{dx}{x^2 + 9}, \quad u = \frac{x}{3}$
22. $\int \frac{9r^2\, dr}{\sqrt{1 - r^3}}, \quad u = 1 - r^3$
23. $\int \left(1 - \cos \frac{t}{2}\right)^2 \sin \frac{t}{2}\, dt, \quad u = 1 - \cos \frac{t}{2}$
24. $\int 8(y^4 + 4y^2 + 1)^2 (y^3 + 2y)\, dy, \quad u = y^4 + 4y^2 + 1$

In Exercises 25–46, use substitution to evaluate the integral.

25. $\int \frac{dx}{(1 - x)^2}$
26. $\int \sec^2 (x + 2)\, dx$
27. $\int \sqrt{\tan x} \sec^2 x\, dx$
28. $\int \sec \left(\theta + \frac{\pi}{2}\right) \tan \left(\theta + \frac{\pi}{2}\right) d\theta$
29. $\int \tan(4x + 2)\, dx$
30. $\int 3(\sin x)^{-2}\, dx$
31. $\int \cos (3z + 4)\, dz$
32. $\int \sqrt{\cot x} \csc^2 x\, dx$
33. $\int \frac{\ln^6 x}{x}\, dx$
34. $\int \tan^7 \left(\frac{x}{2}\right) \sec^2 \left(\frac{x}{2}\right) dx$
35. $\int s^{1/3} \cos (s^{4/3} - 8)\, ds$
36. $\int \frac{dx}{\sin^2 3x}$
37. $\int \frac{\sin (2t + 1)}{\cos^2 (2t + 1)}\, dt$
38. $\int \frac{6 \cos t}{(2 + \sin t)^2}\, dt$
39. $\int \frac{dx}{x \ln x}$
40. $\int \tan^2 x \sec^2 x\, dx$

41. $\displaystyle\int \frac{x\,dx}{x^2+1}$ **42.** $\displaystyle\int \frac{40\,dx}{x^2+25}$

43. $\displaystyle\int \frac{dx}{\cot 3x}$ **44.** $\displaystyle\int \frac{dx}{\sqrt{5x+8}}$

45. $\displaystyle\int \sec x\,dx$ (*Hint:* Multiply the integrand by

$$\frac{\sec x + \tan x}{\sec x + \tan x}$$

and then use a substitution to integrate the result.)

46. $\displaystyle\int \csc x\,dx$ (*Hint:* Multiply the integrand by

$$\frac{\csc x + \cot x}{\csc x + \cot x}$$

and then use a substitution to integrate the result.)

In Exercises 47–52, use the given trigonometric identity to set up a u-substitution and then evaluate the indefinite integral.

47. $\displaystyle\int \sin^3 2x\,dx,\quad \sin^2 2x = 1 - \cos^2 2x$

48. $\displaystyle\int \sec^4 x\,dx,\quad \sec^2 x = 1 + \tan^2 x$

49. $\displaystyle\int 2\sin^2 x\,dx,\quad \cos 2x = 1 - 2\sin^2 x$

50. $\displaystyle\int 4\cos^2 x\,dx,\quad \cos 2x = 2\cos^2 x - 1$

51. $\displaystyle\int \tan^4 x\,dx,\quad \tan^2 x = \sec^2 x - 1$

52. $\displaystyle\int (\cos^4 x - \sin^4 x)\,dx,\quad \cos 2x = \cos^2 x - \sin^2 x$

In Exercises 53–66, make a u-substitution and integrate from $u(a)$ to $u(b)$.

53. $\displaystyle\int_0^3 \sqrt{y+1}\,dy$ **54.** $\displaystyle\int_0^1 r\sqrt{1-r^2}\,dr$

55. $\displaystyle\int_{-\pi/4}^0 \tan x \sec^2 x\,dx$ **56.** $\displaystyle\int_{-1}^1 \frac{5r}{(4+r^2)^2}\,dr$

57. $\displaystyle\int_0^1 \frac{10\sqrt{\theta}}{(1+\theta^{3/2})^2}\,d\theta$ **58.** $\displaystyle\int_{-\pi}^{\pi} \frac{\cos x}{\sqrt{4+3\sin x}}\,dx$

59. $\displaystyle\int_0^1 \sqrt{t^5+2t}\,(5t^4+2)\,dt$ **60.** $\displaystyle\int_0^{\pi/6} \cos^{-3} 2\theta \sin 2\theta\,d\theta$

61. $\displaystyle\int_0^7 \frac{dx}{x+2}$ **62.** $\displaystyle\int_2^5 \frac{dx}{2x-3}$

63. $\displaystyle\int_1^2 \frac{dt}{t-3}$ **64.** $\displaystyle\int_{\pi/4}^{3\pi/4} \cot x\,dx$

65. $\displaystyle\int_{-1}^3 \frac{x\,dx}{x^2+1}$ **66.** $\displaystyle\int_0^2 \frac{e^x\,dx}{3+e^x}$

Two Routes to the Integral In Exercises 67 and 68, make a substitution $u = \cdots$ (an expression in x), $du = \cdots$. Then

(a) integrate with respect to u from $u(a)$ to $u(b)$.

(b) find an antiderivative with respect to u, replace u by the expression in x, then evaluate from a to b.

67. $\displaystyle\int_0^1 \frac{x^3}{\sqrt{x^4+9}}\,dx$

68. $\displaystyle\int_{\pi/6}^{\pi/3} (1-\cos 3x)\sin 3x\,dx$

69. Show that

$$y = \ln\left|\frac{\cos 3}{\cos x}\right| + 5$$

is the solution to the initial value problem

$$\frac{dy}{dx} = \tan x,\quad f(3) = 5.$$

(See the discussion following Example 4, Section 6.4.)

70. Show that

$$y = \ln\left|\frac{\sin x}{\sin 2}\right| + 6$$

is the solution to the initial value problem

$$\frac{dy}{dx} = \cot x,\quad f(2) = 6.$$

Standardized Test Questions

71. True or False By u-substitution, $\int_0^{\pi/4} \tan^3 x \sec^2 x\,dx = \int_0^{\pi/4} u^3\,du$. Justify your answer.

72. True or False If f is positive and differentiable on $[a, b]$, then $\displaystyle\int_a^b \frac{f'(x)dx}{f(x)} = \ln\left(\frac{f(b)}{f(a)}\right)$. Justify your answer.

73. Multiple Choice $\int \tan x\,dx =$

(A) $\dfrac{\tan^2 x}{2} + C$ (B) $\ln|\cot x| + C$ (C) $\ln|\cos x| + C$

(D) $-\ln|\cos x| + C$ (E) $-\ln|\cot x| + C$

74. Multiple Choice $\int_0^2 e^{2x}\,dx =$

(A) $\dfrac{e^4}{2}$ (B) $e^4 - 1$ (C) $e^4 - 2$ (D) $2e^4 - 2$ (E) $\dfrac{e^4-1}{2}$

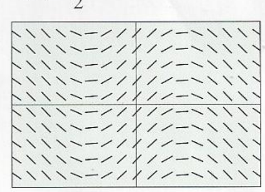
75. Multiple Choice If $\int_3^5 f(x-a)\,dx = 7$ where a is a constant, then $\int_{3-a}^{5-a} f(x)\,dx =$

(A) $7 + a$ (B) 7 (C) $7 - a$ (D) $a - 7$ (E) -7

76. Multiple Choice If the differential equation $dy/dx = f(x)$ leads to the slope field shown below, which of the following could be $\int f(x)\,dx$?

(A) $\sin x + C$ (B) $\cos x + C$ (C) $-\sin x + C$

(D) $-\cos x + C$ (E) $\dfrac{\sin^2 x}{2} + C$



Explorations

77. *Constant of Integration* Consider the integral

$$\int \sqrt{x+1}\, dx.$$

(a) Show that $\int \sqrt{x+1}\, dx = \frac{2}{3}(x+1)^{3/2} + C.$

(b) Writing to Learn Explain why

$$y_1 = \int_0^x \sqrt{t+1}\, dt \quad \text{and} \quad y_2 = \int_3^x \sqrt{t+1}\, dt$$

are antiderivatives of $\sqrt{x+1}$.

(c) Use a table of values for $y_1 - y_2$ to find the value of C for which $y_1 = y_2 + C$.

(d) Writing to Learn Give a convincing argument that

$$C = \int_0^3 \sqrt{x+1}\, dx.$$

78. Group Activity *Making Connections* Suppose that

$$\int f(x)\, dx = F(x) + C.$$

(a) Explain how you can use the derivative of $F(x) + C$ to confirm the integration is correct.

(b) Explain how you can use a slope field for $dy/dx = f(x)$ and the graph of $y = F(x)$ to support your evaluation of the integral.

(c) Explain how you can use the graphs of $y_1 = F(x)$ and $y_2 = \int_0^x f(t)\, dt$ to support your evaluation of the integral.

(d) Explain how you can use a table of values for $y_1 - y_2$, y_1 and y_2 defined as in part (c), to support your evaluation of the integral.

(e) Explain how you can use graphs of f and NDER of $F(x)$ to support your evaluation of the integral.

(f) Illustrate parts (a)–(e) for $f(x) = \dfrac{x}{\sqrt{x^2+1}}$.

79. *Different Solutions?* Consider the integral $\int 2 \sin x \cos x\, dx$.

(a) Evaluate the integral using the substitution $u = \sin x$.

(b) Evaluate the integral using the substitution $u = \cos x$.

(c) Writing to Learn Explain why the different-looking answers in parts (a) and (b) are actually equivalent.

80. *Different Solutions?* Consider the integral $\int 2 \sec^2 x \tan x\, dx$.

(a) Evaluate the integral using the substitution $u = \tan x$.

(b) Evaluate the integral using the substitution $u = \sec x$.

(c) Writing to Learn Explain why the different-looking answers in parts (a) and (b) are actually equivalent.

Extending the Ideas

81. *Trigonometric Substitution* Suppose $u = \sin^{-1} x$. Then $\cos u > 0$.

(a) Use the substitution $x = \sin u$, $dx = \cos u\, du$ to show that

$$\int \frac{dx}{\sqrt{1-x^2}} = \int 1\, du.$$

(b) Evaluate $\int 1\, du$ to show that $\int \dfrac{dx}{\sqrt{1-x^2}} = \sin^{-1} x + C.$

82. *Trigonometric Substitution* Suppose $u = \tan^{-1} x$.

(a) Use the substitution $x = \tan u$, $dx = \sec^2 u\, du$ to show that

$$\int \frac{dx}{1+x^2} = \int 1\, du.$$

(b) Evaluate $\int 1\, du$ to show that $\int \dfrac{dx}{1+x^2} = \tan^{-1} x + C.$

83. *Trigonometric Substitution* Suppose $\sqrt{x} = \sin y$.

(a) Use the substitution $x = \sin^2 y$, $dx = 2 \sin y \cos y\, dy$ to show that

$$\int_0^{1/2} \frac{\sqrt{x}\, dx}{\sqrt{1-x}} = \int_0^{\pi/4} 2 \sin^2 y\, dy.$$

(b) Use the identity given in Exercise 49 to evaluate the definite integral without a calculator.

84. *Trigonometric Substitution* Suppose $u = \tan^{-1} x$.

(a) Use the substitution $x = \tan u$, $dx = \sec^2 u\, du$ to show that

$$\int_0^{\sqrt{3}} \frac{dx}{\sqrt{1+x^2}} = \int_0^{\pi/3} \sec u\, du.$$

(b) Use the hint in Exercise 45 to evaluate the definite integral without a calculator.