AP CALCULUS AB

SEPARABLE DIFFERENTIAL EQUATIONS

1) Solve

$$\frac{dy}{dx} = -\frac{y}{2\sqrt{x}}$$

for $y = y(x)$ subject to the initial condition $y(0) = 4$.

$\frac{dy}{y} = -\frac{dx}{2\sqrt{x}}$, $\int_4^y \frac{dz}{z} = -\frac{1}{2}\int_0^x \frac{dt}{\sqrt{t}}$,

$[\ln z]_4^y = -\frac{1}{\cancel{2}}[\cancel{2}\sqrt{t}]_0^x$

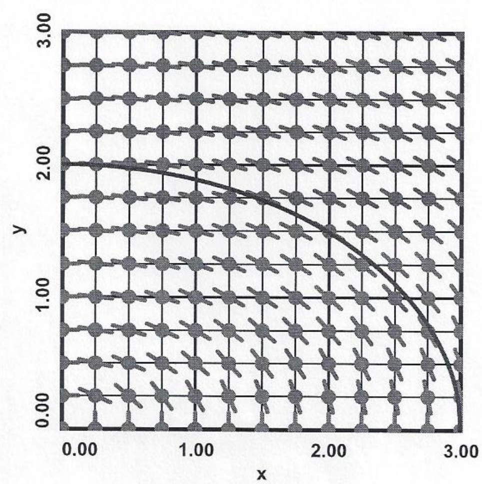
$\ln y - \ln 4 = -\sqrt{x}$, $\ln\left(\frac{y}{4}\right) = -\sqrt{x}$,

$\frac{y}{4} = e^{-\sqrt{x}}$, $y = 4e^{-\sqrt{x}}$

2) Solve

$$\frac{dy}{dx} = -\frac{4x}{9y}$$

subject to the initial condition $y(0) = 2$. (*Hint:* the solution is an ellipse). Also, graph the solution $y = y(x)$ on the slope field provided.



$9y\,dy = -4x\,dx$, $9\int_2^y z\,dz = -4\int_0^x t\,dt$, $9\left[\frac{z^2}{\cancel{2}}\right]_2^y = -4\left[\frac{t^2}{\cancel{2}}\right]_0^x$,

$9(y^2-4) = -4x^2$, $4x^2+9y^2 = 36 = 4\cdot 9$, $\frac{x^2}{9}+\frac{y^2}{4} = 1$, $\frac{x^2}{3^2}+\frac{y^2}{2^2} = 1$

an ellipse

First quadrant, solve for y ... $9y^2 = 4\cdot 9 - 4x^2 = 4(9-x^2)$,

$y^2 = \frac{4}{9}(9-x^2)$, $y = \frac{2}{3}\sqrt{9-x^2}$