7.1. Separable Differential Equations

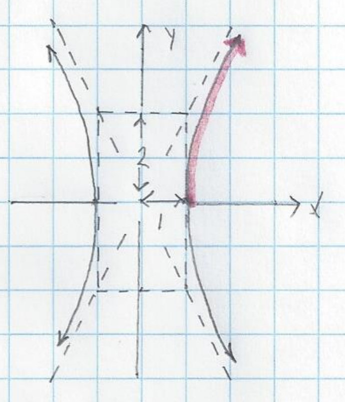
Can put all the x's on one side of the equation, and all the y's on the other side...

Example #1. Solve $\frac{dy}{dx} = \frac{4x}{y}$ for $y = y(x)$ subject to the initial condition $y(1) = 0$.

SOLUTION: $y\,dy = 4x\,dx$. Integrate from the initial state to some later state $\Rightarrow$ $\int_0^y z\,dz = 4\int_1^x t\,dt$, $\left[\frac{z^2}{\cancel{2}}\right]_0^y = 4\left[\frac{t^2}{\cancel{2}}\right]_1^x$,

$y^2 = 4(x^2-1)$, $y = \pm 2\sqrt{x^2-1}$ ◄ This is actually the equation of a hyperbola... $y^2 = 4x^2 - 4$, $4x^2 - y^2 = 4$, $x^2 - \frac{y^2}{4} = 1$,

$\frac{x^2}{1^2} - \frac{y^2}{2^2} = 1$

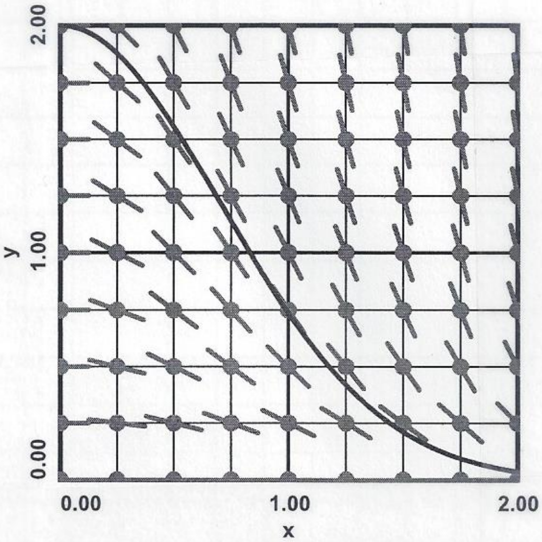


If you just want the graph in the first quadrant, then $y = 2\sqrt{x^2-1}$, $x \in [1, \infty)$ ◄

Example #2. Solve $\frac{dy}{dx} = -2xy$ for $y = y(x)$ subject to the initial condition $y(0) = 2$. Also graph $y = y(x)$ on $x \in [0,2]$ and $y \in [0,2]$ along with its slope field.

SOLUTION: $\frac{dy}{y} = -2x\,dx$. Integrate from the initial state to some later state $\Rightarrow$ $\int_2^y \frac{dz}{z} = -2\int_0^x t\,dt$, $\left[\ln z\right]_2^y = -\cancel{2}\left[\frac{t^2}{\cancel{2}}\right]_0^x$,

$\ln y - \ln 2 = -x^2$, $\ln\left(\frac{y}{2}\right) = -x^2$, $\frac{y}{2} = e^{-x^2}$, $y = 2e^{-x^2}$ ◄


2.00
1.00
0.00
y
0.00
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2.00
x