AP CALCULUS AB

THE SIMPLE FIRST-ORDER LINEAR DIFFERENTIAL EQUATION

For problems **1** and **2**, verify the indefinite integrals by differentiation.

1)

$$\int e^{-x}\sin x\,dx = \int f'(x)\,dx =$$
$$= -\frac{1}{2}e^{-x}(\sin x + \cos x) = f(x)$$

$$f'(x) = -\frac{1}{2}\left[-e^{-x}(\sin x + \cos x) + e^{-x}(\cos x - \sin x)\right] =$$

$$= -\frac{1}{2}e^{-x}(-\sin x - \cancel{\cos x} + \cancel{\cos x} - \sin x)$$

$$= -\frac{1}{\cancel{2}}e^{-x}(-\cancel{2}\sin x) =$$

$$= e^{-x}\sin x$$

2)

$$\int x^2e^x\,dx = \int f'(x)\,dx$$
$$= x^2e^x - 2xe^x + 2e^x = f(x)$$

$$f'(x) = 2xe^x + x^2e^x - 2(e^x + xe^x) + 2e^x =$$

$$= \cancel{2xe^x} + x^2e^x - \cancel{2e^x} - \cancel{2xe^x} + \cancel{2e^x} =$$

$$= x^2e^x$$

3) Solve

$$\frac{dy}{dx} + y = x^2$$

for $y = y(x)$ subject to the initial condition $y(1) = 2$. *Hint:* $k = 1$. Also, you will need to use the integral from problem **2**.

$$e^x\frac{dy}{dx} + e^xy = e^xx^2$$

$$u = e^xy \qquad \frac{du}{dx} = e^xy + e^x\frac{dy}{dx}$$

$$\frac{du}{dx} = e^xx^2, \qquad du = x^2e^x\,dx$$

$$y(1) = 2 \Rightarrow u_0 = e^1\cdot 2 = 2e,$$

$$\int_{u_0}^{u} dv = [v]_{u_0}^{u} = [v]_{2e}^{e^xy} = \int_1^x t^2e^t\,dt$$

$$e^xy - 2e = \left[t^2e^t - 2te^t + 2e^t\right]_1^x =$$

$$= x^2e^x - 2xe^x + 2e^x - e$$

$$e^xy = e + e^x(x^2 - 2x + 2)$$

$$y = e\cdot e^{-x} + x^2 - 2x + 2$$

$$y = e^{1-x} + x^2 - 2x + 2$$