7.2. Substitution in Indefinite Integrals

Examples. Use the indicated substitutions to evaluate the integrals.

(1) $\int \cos\left(\frac{\pi x}{2}\right) dx$, $u = \frac{\pi x}{2}$

$\frac{du}{dx} = \frac{\pi}{2}$, $du = \frac{\pi}{2} dx$, $dx = \frac{2}{\pi} du$, $\int \cos\left(\frac{\pi x}{2}\right) dx = \int \cos u \cdot \frac{2}{\pi} du = \frac{2}{\pi} \int \cos u \, du =$

$= \frac{2}{\pi} \sin u + k = \frac{2}{\pi} \sin\left(\frac{\pi x}{2}\right) + k$

(2) $\int x \sin(x^2) dx$, $u = x^2$

$\frac{du}{dx} = 2x$, $du = 2x\,dx$, $dx = \frac{du}{2x}$, $\int x \sin(x^2) dx = \int \cancel{x} \sin u \cdot \frac{du}{2\cancel{x}} = \frac{1}{2} \int \sin u \, du =$

$= \frac{1}{2} \cdot -\cos u + k = -\frac{1}{2} \cos(x^2) + k$

(3) $\int \frac{x}{\sqrt{a^2 - x^2}} dx$, $u = a^2 - x^2$

$\frac{du}{dx} = -2x$, $du = -2x\,dx$, $dx = -\frac{du}{2x}$, $\int \frac{x}{\sqrt{a^2 - x^2}} dx = \int \frac{\cancel{x}}{\sqrt{u}} \cdot -\frac{du}{2\cancel{x}} =$

$= -\frac{1}{2} \int \frac{du}{\sqrt{u}} = -\frac{1}{\cancel{2}} \cdot \cancel{2} \sqrt{u} + k = -\sqrt{a^2 - x^2} + k$

(4) $\int \cot x \, dx$, $u = \sin x$

$\frac{du}{dx} = \cos x$, $du = \cos x \, dx$, $dx = \frac{du}{\cos x}$, $\int \cot x \, dx = \int \frac{\cos x}{\sin x} dx = \int \frac{\cancel{\cos x}}{u} \cdot \frac{du}{\cancel{\cos x}} =$

$= \int \frac{du}{u} = \ln|u| + k = \ln|\sin x| + k$

(5) $\int \csc x \, dx$, $u = \csc x - \cot x$

$\frac{du}{dx} = -\csc x \cot x + \csc^2 x = \csc x (\csc x - \cot x) = \csc x \cdot u$, $du = u \csc x \, dx$,

$dx = \frac{du}{u \csc x}$, $\int \csc x \, dx = \int \cancel{\csc x} \cdot \frac{du}{u \cancel{\csc x}} = \int \frac{du}{u} = \ln|u| + k = \ln|\csc x - \cot x| + k$

7.2. Substitution in Indefinite Integrals

CLASS WORK Use the indicated substitutions to evaluate the integrals.

(1) $\int \sin\left(\frac{\pi x}{8}\right) dx$, $u = \frac{\pi x}{8}$

(2) $\int x \sec^2(x^2)\, dx$, $u = x^2$

(3) $\int x\sqrt{a^2 - x^2}\, dx$, $u = a^2 - x^2$

(4) $\int \tan x\, dx$, $u = \cos x$

(5) $\int \sec x\, dx$, $u = \sec x + \tan x$

SOLUTIONS

(1) $\frac{du}{dx} = \frac{\pi}{8}$, $du = \frac{\pi}{8}dx$, $dx = \frac{8}{\pi}du$, $\int \sin\left(\frac{\pi x}{8}\right)dx = \int \sin u \cdot \frac{8}{\pi}du = \frac{8}{\pi}\int \sin u\, du =$

$= \frac{8}{\pi}\cdot -\cos u + k = -\frac{8}{\pi}\cos\left(\frac{\pi x}{8}\right) + k$ $\blacktriangleleft$

(2) $\frac{du}{dx} = 2x$, $du = 2x\,dx$, $dx = \frac{du}{2x}$, $\int x\sec^2(x^2)\,dx = \int \cancel{x}\sec^2 u \cdot \frac{du}{2\cancel{x}} =$

$= \frac{1}{2}\int \sec^2 u\, du = \frac{1}{2}\tan u + k = \frac{1}{2}\tan(x^2) + k$ $\blacktriangleleft$

(3) $\frac{du}{dx} = -2x$, $du = -2x\,dx$, $dx = -\frac{du}{2x}$, $\int x\sqrt{a^2-x^2}\,dx = \int \cancel{x}\sqrt{u}\cdot -\frac{du}{2\cancel{x}} =$

$= -\frac{1}{2}\int \sqrt{u}\,du = -\frac{1}{\cancel{2}}\cdot\frac{\cancel{2}}{3}\sqrt{u^3} + k = -\frac{1}{3}\sqrt{(a^2-x^2)^3} + k$ $\blacktriangleleft$

(4) $\frac{du}{dx} = -\sin x$, $du = -\sin x\,dx$, $dx = -\frac{1\,du}{\sin x}$ $\int \tan x\,dx = \int \frac{\sin x}{\cos x}dx =$

$= \int \frac{\cancel{\sin x}}{-u}\cdot\frac{-du}{\cancel{\sin x}} = -\int\frac{du}{u} = -\ln|u| + k = \ln\left|\frac{1}{u}\right| + k = \ln\left|\frac{1}{\cos x}\right| + k =$

$= \ln|\sec x| + k$ $\blacktriangleleft$

(5) $\frac{du}{dx} = \sec x\tan x + \sec^2 x = \sec x(\sec x + \tan x) = \sec x\cdot u$, $du = u\sec x\,dx$,

$dx = \frac{du}{u\sec x}$, $\int \sec x\,dx = \int \cancel{\sec x}\cdot\frac{du}{u\cancel{\sec x}} = \int\frac{du}{u} = \ln|u| + k = \ln|\sec x + \tan x| + k$ $\blacktriangleleft$