Pg. 335

7.1. Exact Differential Equations

(12) $\frac{dy}{dx} = 2e^x - \cos x$, $y(0) = 3$, $dy = (2e^x - \cos x)\,dx$, $\int_3^y dz = \int_0^x (2e^t - \cos t)\,dt$,

$[z]_3^y = [2e^t - \sin t]_0^x$, $y - 3 = 2e^x - \sin x - 2$, $y = 2e^x - \sin x + 1$ ◂

(13) $\frac{dy}{dx} = 7x^6 - 3x^2 + 5$, $y(1) = 1$, $dy = (7x^6 - 3x^2 + 5)\,dx$, $\int_1^y dz = \int_1^x (7t^6 - 3t^2 + 5)\,dt$,

$[z]_1^y = [t^7 - t^3 + 5t]_1^x$, $y - 1 = x^7 - x^3 + 5x - 5$, $y = x^7 - x^3 + 5x - 4$ ◂

(78) (a) $\frac{d^2y}{dx^2} = 24x^2 - 10$, $y'(1) = 3$, $y(1) = 5$, $dy' = (24x^2 - 10)\,dx$,

$\int_3^{y'} dz = \int_1^x (24t^2 - 10)\,dt$, $[z]_3^{y'} = [8t^3 - 10t]_1^x$, $y' - 3 = 8x^3 - 10x + 2$

$y' = \frac{dy}{dx} = 8x^3 - 10x + 5$, $dy = (8x^3 - 10x + 5)\,dx$, $\int_5^y dz = \int_1^x (8t^3 - 10t + 5)\,dt$,

$[z]_5^y = [2t^4 - 5t^2 + 5t]_1^x$, $y - 5 = 2x^4 - 5x^2 + 5x - 2$, $y = 2x^4 - 5x^2 + 5x + 3$ ◂

(b) $\frac{d^2y}{dx^2} = \cos x - \sin x$, $y'(0) = 2$, $y(0) = 0$, $dy' = (\cos x - \sin x)\,dx$,

$\int_2^{y'} dz = \int_0^x (\cos t - \sin t)\,dt$, $[z]_2^{y'} = [\sin t + \cos t]_0^x$, $y' - 2 = \sin x + \cos x - 1$,

$y' = \frac{dy}{dx} = \sin x + \cos x + 1$, $dy = (\sin x + \cos x + 1)\,dx$, $\int_0^y dz = \int_0^x (\sin t + \cos t + 1)\,dt$,

$[z]_0^y = [-\cos t + \sin t + t]_0^x$, $y = -\cos x + \sin x + x + 1$ ◂

Pg. 335

7.1. Slope Fields

(35) c ◂ (38) d ◂

(36) e ◂ (39) b ◂

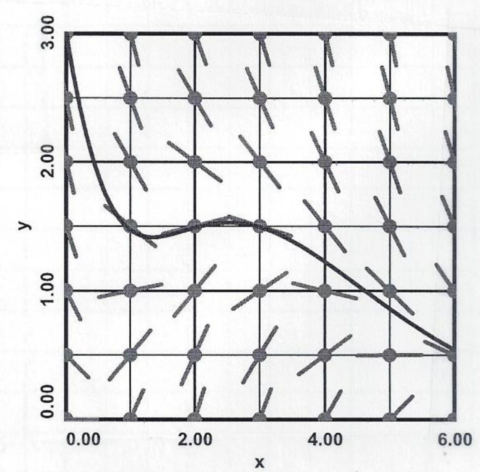
(37) a ◂ (40) f ◂

(1) $\frac{dy}{dx} = 3x^2e^{-x} - y$

(a)

	0	1	2	3	4	5	6	x
3.0	-3.000	-1.896	-1.376	-1.656	-2.121	-2.495	-2.732	
2.5	-2.500	-1.396	-0.876	-1.156	-1.621	-1.995	-2.232	
2.0	-2.000	-0.896	-0.376	-0.656	-1.121	-1.495	-1.732	
1.5	-1.500	-0.396	0.124	-0.156	-0.621	-0.995	-1.232	
1.0	-1.000	0.104	0.624	0.344	-0.121	-0.495	-0.732	
0.5	-0.500	0.604	1.124	0.844	0.379	0.005	-0.232	
0.0	0.000	1.104	1.624	1.344	0.879	0.505	0.268	
y								

(b)



7.1. Separable Differential Equations

pg. 364

(1) $\frac{dy}{dx} = \frac{x}{y}$, $y(1) = 2$, $y\,dy = x\,dx$

$\int_2^y z\,dz = \int_1^x t\,dt$, $\left[\frac{z^2}{\cancel{2}}\right]_2^y = \left[\frac{t^2}{\cancel{2}}\right]_1^x$,

$y^2 - 4 = x^2 - 1$, $y^2 = x^2 + 3$,

$y = \pm\sqrt{x^2+3}$ ◄

(2) $\frac{dy}{dx} = -\frac{x}{y}$, $y(4) = 3$, $y\,dy = -x\,dx$,

$\int_3^y z\,dz = -\int_4^x t\,dt$, $\left[\frac{z^2}{\cancel{2}}\right]_3^y = -\left[\frac{t^2}{\cancel{2}}\right]_4^x$, $y^2 - 9 = -x^2 + 16$, $y^2 = 25 - x^2$,

$y = \pm\sqrt{25 - x^2}$ ◄

(6) $\frac{dy}{dx} = \cos^2 y$, $y(0) = 0$, $\frac{dy}{\cos^2 y} = dx$, $\sec^2 y\,dy = dx$, $\int_0^y \sec^2 z\,dz = \int_0^x dt$,

$\left[\tan z\right]_0^y = \left[t\right]_0^x$, $\tan y = x$, $y = \tan^{-1} x$ ◄

(9) $\frac{dy}{dx} = -2xy^2$, $y(1) = 0.25$, $\frac{dy}{y^2} = -2x\,dx$, $\int_{0.25}^{y} \frac{dz}{z^2} = -2\int_{1}^{x} t\,dt$,

$\left[\cancel{-}\frac{1}{z}\right]_{0.25}^{y} = \cancel{-}\cancel{2}\left[\frac{t^2}{\cancel{2}}\right]_1^x$, $\frac{1}{y} - 4 = x^2 - 1$, $\frac{1}{y} = x^2 + 3$, $y = \frac{1}{x^2+3}$

(43) $\frac{dV}{dt} = -\frac{1}{40}V$, $\frac{dV}{V} = -\frac{1}{40}dt$, $\int_{V_0}^{V} \frac{dz}{z} = -\frac{1}{40}\int_0^t dx$, $\left[\ln z\right]_{V_0}^{V} = -\frac{1}{40}\left[x\right]_0^t$,

(a) $\ln V - \ln V_0 = \ln\left(\frac{V}{V_0}\right) = -\frac{1}{40}t$, $\frac{V}{V_0} = e^{-t/40}$, $V = V_0 e^{-t/40}$

(b) $0.1\cancel{V_0} = \cancel{V_0}e^{-t/40}$, $-\frac{t}{40} = \ln 0.1$, $t = -40\ln 0.1 = 92.10$ sec

Supplemental

7.1. The Simple First-Order Linear Differential Equation

(2) $\frac{dy}{dx} + y = 3x^2e^{-x}$, $y(0) = 3$, $e^x\frac{dy}{dx} + e^xy = 3x^2$, Let $u = e^xy$,

(a) $\frac{du}{dx} = e^xy + e^x\frac{dy}{dx}$, $\frac{du}{dx} = 3x^2$, $du = 3x^2dx$, $\int_{u_0}^{u} dz = 3\int_0^x t^2dt$,

$[z]_{u_0}^{u} = [z]_3^{e^xy} = \cancel{3}\left[\frac{t^3}{\cancel{3}}\right]_0^x$, $e^xy - 3 = x^3$, $e^xy = x^3+3$, $y = (x^3+3)e^{-x}$

(b) See page 2

7.2. Substitution in the Indefinite Integral

pg. 346

(17) $\int \sin 3x\,dx$, $u = 3x$, $\frac{du}{dx} = 3$, $dx = \frac{du}{3}$, $\int \sin 3x\,dx = \int \sin u \cdot \frac{du}{3} = \frac{1}{3}\int \sin u\,du =$

$= -\frac{1}{3}\cos u + k = -\frac{1}{3}\cos 3x + k$

(18) $\int x\cos(2x^2)\,dx$, $u = 2x^2$, $\frac{du}{dx} = 4x$, $du = 4x\,dx$, $dx = \frac{du}{4x}$,

$\int x\cos(2x^2)\,dx = \int \cancel{x}\cos u \cdot \frac{du}{4\cancel{x}} = \frac{1}{4}\int \cos u\,du = \frac{1}{4}\sin u + k = \frac{1}{4}\sin(2x^2) + k$

(25) $\int \frac{dx}{(1-x)^2}$, $u = 1-x$, $\frac{du}{dx} = -1$, $dx = -du$, $\int \frac{dx}{(1-x)^2} = \int \frac{-du}{u^2} = -\int \frac{du}{u^2} =$

$= -\,-\frac{1}{u} + k = \frac{1}{1-x} + k$

(39) $\int \frac{dx}{x\ln x}$, $u = \ln x$, $\frac{du}{dx} = \frac{1}{x}$, $du = \frac{dx}{x}$, $\int \frac{dx}{x\ln x} = \int \frac{du}{u} = \ln u + k =$

$= \ln\left[\ln x\right] + k$

(41) $\int \frac{x\,dx}{x^2+1}$, $u=x^2+1$, $\frac{du}{dx}=2x$, $du=2x\,dx$, $dx=\frac{du}{2x}$, $\int \frac{x\,dx}{x^2+1} = \int \frac{\cancel{x}}{u}\cdot\frac{du}{2\cancel{x}} =$

$= \frac{1}{2}\int \frac{du}{u} = \frac{1}{2}\ln u + k = \ln\sqrt{u} + k = \ln\sqrt{x^2+1} + k$ ◄

(47) $\int \sin^3 2x\,dx = \int \sin 2x \cdot \sin^2 2x\,dx = \int \sin 2x\,[1-\cos^2 2x]\,dx =$

$= \int \sin 2x\,dx - \int \sin 2x \cos^2 2x\,dx$, $\int \sin 2x\,dx$, $u=2x$, $du=2dx$, $dx=\frac{du}{2}$,

$\int \sin 2x\,dx = \int \sin u \cdot \frac{du}{2} = \frac{1}{2}\int \sin u\,du = -\frac{1}{2}\cos u = -\frac{1}{2}\cos 2x$, $\int \sin 2x \cos^2 2x\,dx$,

$v=\cos 2x$, $\frac{dv}{dx} = -\sin 2x \cdot 2$, $dv = -2\sin 2x\,dx$, $dx = \frac{dv}{-2\sin 2x}$, $\int \sin 2x \cos^2 2x\,dx =$

$= \int \cancel{\sin 2x}\,v^2 \cdot \frac{dv}{-2\cancel{\sin 2x}} = -\frac{1}{2}\int v^2\,dv = -\frac{1}{2}\cdot\frac{v^3}{3} = -\frac{1}{6}v^3 = -\frac{1}{6}\cos^3 2x$

$\int \sin^3 2x\,dx = -\frac{1}{2}\cos 2x + \frac{1}{6}\cos^3 2x + k$ ◄

Supplemental

7.2. Substitution in the Definite Integral

(3) $\int_0^3 x\sqrt{9-x^2}\,dx$, $u=9-x^2$, $x=0 \to u=9$, $x=3 \to u=0$, $\frac{du}{dx}=-2x$, $du=-2x\,dx$,

$dx=\frac{du}{-2x}$, $\int_{x=0}^{x=3} x\sqrt{9-x^2}\,dx = \int_{u=9}^{u=0} \cancel{x}\sqrt{u}\cdot\frac{du}{-2\cancel{x}} = \frac{1}{2}\int_0^9 \sqrt{u}\,du = \frac{1}{\cancel{2}}\left[\frac{\cancel{2}}{3}\sqrt{u^3}\right]_0^9 = 9$ ◄

(4) $\int_0^1 \frac{x}{\sqrt{4-x^2}}\,dx$, $u=4-x^2$, $x=0 \to u=4$, $x=1 \to u=3$, $\frac{du}{dx}=-2x$, $du=-2x\,dx$,

$dx=-\frac{du}{2x}$, $\int_{x=0}^{x=1} \frac{x}{\sqrt{4-x^2}}\,dx = \int_{u=4}^{u=3} \frac{\cancel{x}}{\sqrt{u}}\cdot -\frac{du}{2\cancel{x}} = \frac{1}{2}\int_3^4 \frac{du}{\sqrt{u}} = \frac{1}{\cancel{2}}\left[\cancel{2}\sqrt{u}\right]_3^4 = 2-\sqrt{3} =$

$= 0.267\ 949\ 1924$ ◄

(5) $\int_1^5 \frac{\ln x}{x}\,dx$, $u=\ln x$, $\frac{du}{dx}=\frac{1}{x}$, $du=\frac{dx}{x}$, $x=1 \to u=0$, $x=5 \to u=\ln 5$,

$\int_{x=1}^{x=5} \frac{\ln x}{x}\,dx = \int_{u=0}^{u=\ln 5} u\,du = \left[\frac{u^2}{2}\right]_0^{\ln 5} = \frac{1}{2}(\ln 5)^2 = 1.295\ 145\ 197$ ◄

(6) $\int_0^{\sqrt{\pi/2}} x\cos(x^2)\,dx$, $u=x^2$, $du=2x\,dx$, $dx=\frac{du}{2x}$, $x=0 \to u=0$, $x=\sqrt{\pi/2} \to u=\pi/4$

$\int_{x=0}^{x=\sqrt{\pi/2}} x\cos(x^2)\,dx = \int_{u=0}^{u=\pi/4} \cancel{x}\cos u \cdot \frac{du}{2\cancel{x}} = \frac{1}{2}\int_0^{\pi/4} \cos u\,du = \frac{1}{2}\left[\sin u\right]_0^{\pi/4} = \frac{\sqrt{2}}{4} = 0.353\ 553\ 3906$ ◄