Quiz #11 Study Guide

(1) $\frac{dy}{dx} = \sin x - 4x^3$, $dy = (\sin x - 4x^3)\,dx$, $\int_7^y dz = \int_0^x (\sin t - 4t^3)\,dt$,

$[z]_7^y = [-\cos t - t^4]_0^x$, $y - 7 = -\cos x - x^4 + 1$, $y = -\cos x - x^4 + 8$

(2) $\frac{dy'}{dx} = 5x + 3$, $dy' = (5x+3)\,dx$, $\int_{-4}^{y'} dz = \int_0^x (5t+3)\,dt$, $[z]_{-4}^{y'} = \left[\frac{5}{2}t^2 + 3t\right]_0^x$,

$y' + 4 = \frac{5}{2}x^2 + 3x$, $\frac{dy}{dx} = \frac{5}{2}x^2 + 3x - 4$, $dy = \left(\frac{5}{2}x^2 + 3x - 4\right)dx$,

$\int_3^y dz = \int_0^x \left(\frac{5}{2}t^2 + 3t - 4\right)dt$, $[z]_3^y = \left[\frac{5}{6}t^3 + \frac{3}{2}t^2 - 4t\right]_0^x$, $y - 3 = \frac{5}{6}x^3 + \frac{3}{2}x^2 - 4x$,

$y = \frac{5}{6}x^3 + \frac{3}{2}x^2 - 4x + 3$

(7) $\frac{dy}{y} = -\frac{dx}{2\sqrt{x}}$, $\int_6^y \frac{dz}{z} = -\frac{1}{2}\int_0^x \frac{dt}{\sqrt{t}}$, $[\ln z]_6^y = -\frac{1}{2}\left[2\sqrt{t}\right]_0^x$, $\ln\left(\frac{y}{6}\right) = -\sqrt{x}$,

$\frac{y}{6} = e^{-\sqrt{x}}$, $y = 6e^{-\sqrt{x}}$

(8) $e^{-5x}\frac{dy}{dx} - 5e^{-5x}y = 3x^2$, $u = e^{-5x}y$, $\frac{du}{dx} = -5e^{-5x}y + e^{-5x}\frac{dy}{dx}$,

$\frac{du}{dx} = 3x^2$, $du = 3x^2\,dx$, $\int_{u_0}^{u} dv = 3\int_0^x t^2\,dt$, $u_0 = e^{-5\cdot 0}\cdot 2 = 2$

$[v]_2^{e^{-5x}y} = 3\left[\frac{t^3}{3}\right]_0^x$, $e^{-5x}y - 2 = x^3$, $e^{-5x}y = x^3 + 2$, $y = (x^3+2)e^{5x}$

(9) $\int \frac{dx}{x\ln x}$, $u = \ln x$, $\frac{du}{dx} = \frac{1}{x}$, $du = \frac{dx}{x}$, $\int \frac{dx}{x\ln x} = \int \frac{du}{u} = \ln u + k = \ln[\ln x] + k$

(10) $\int x^2 \sec(4x^3)\tan(4x^3)\,dx$, $u = 4x^3$, $\frac{du}{dx} = 12x^2$, $dx = \frac{du}{12x^2}$,

$\int x^2 \sec(4x^3)\tan(4x^3)\,dx = \int x^2 \sec u \tan u \frac{du}{12x^2} = \frac{1}{12}\int \sec u \tan u\,du =$

$= \frac{1}{12}\sec u + k = \frac{1}{12}\sec(4x^3) + k$

(11) $\int_0^{\pi/6} \sin x \sec(\cos x)\tan(\cos x)\,dx$, $u = \cos x$, $\frac{du}{dx} = -\sin x$, $dx = -\frac{du}{\sin x}$

$x = 0 \Rightarrow u = 1$, $x = \frac{\pi}{6} \to u = \frac{\sqrt{3}}{2}$, $\int_0^{\pi/6} \sin x \sec(\cos x)\tan(\cos x)\,dx =$

$$= \int_1^{\sqrt{3}/2} \sin x \sec u \tan u \cdot -\frac{du}{\sin x} = \int_{\sqrt{3}/2}^{1} \sec u \tan u \, du = [\sec u]_{\sqrt{3}/2}^{1} =$$

$$= \sec 1 - \sec \frac{\sqrt{3}}{2} = 0.307\,270\,7986$$

(12) $\int_0^{\pi/3} \sec^2 x \tan x \sin(\sec^2 x)\, dx$, $u = \sec^2 x$, $\frac{du}{dx} = 2\sec x \sec x \tan x = 2\sec^2 x \tan x$,

$dx = \frac{du}{2\sec^2 x \tan x}$ $\quad x = 0 \Rightarrow u = 1$, $x = \frac{\pi}{3} \to u = 4$

$$\int_0^{\pi/3} \sec^2 x \tan x \sin(\sec^2 x)\, dx = \int_1^4 \sec^2 x \tan x \sin u \cdot \frac{du}{2\sec^2 x \tan x} =$$

$$= \frac{1}{2}\int_1^4 \sin u \, du = \frac{1}{2}[-\cos u]_1^4 = \frac{1}{2}[\cos 1 - \cos 4] = 0.596\,972\,9634$$